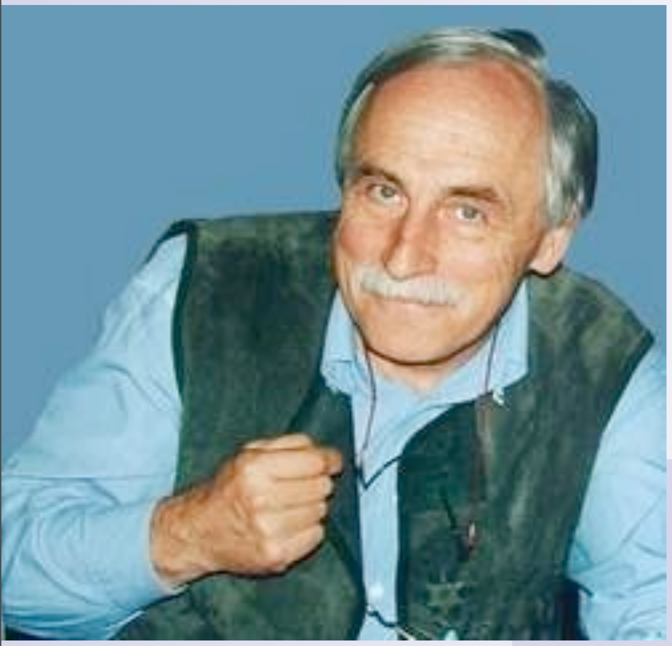


Exploring the Wonderland of the QCD Phase Diagram

Berndt Mueller

The Berkeley School of Collective Dynamics in High Energy Collisions

LBNL: June 7-11, 2010

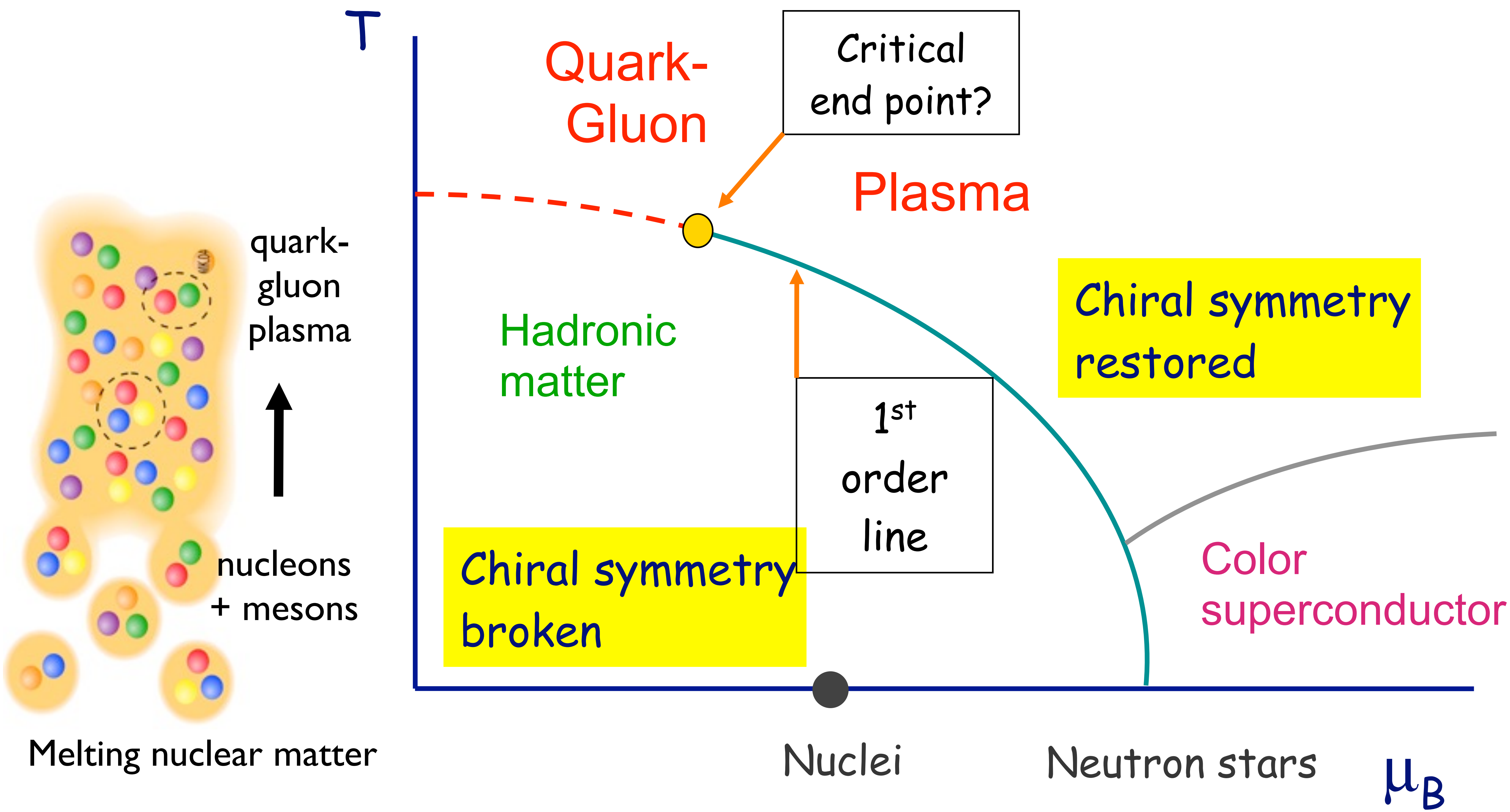
When you give the final lecture,
there’s nothing much left to say ...

When you give the final lecture,
there's nothing much left to say ...

... so, following in the footsteps of Reinhard Stock,
it may be worthwhile to take a look at the

BIG PICTURE.

RHIC in the QCD Landscape



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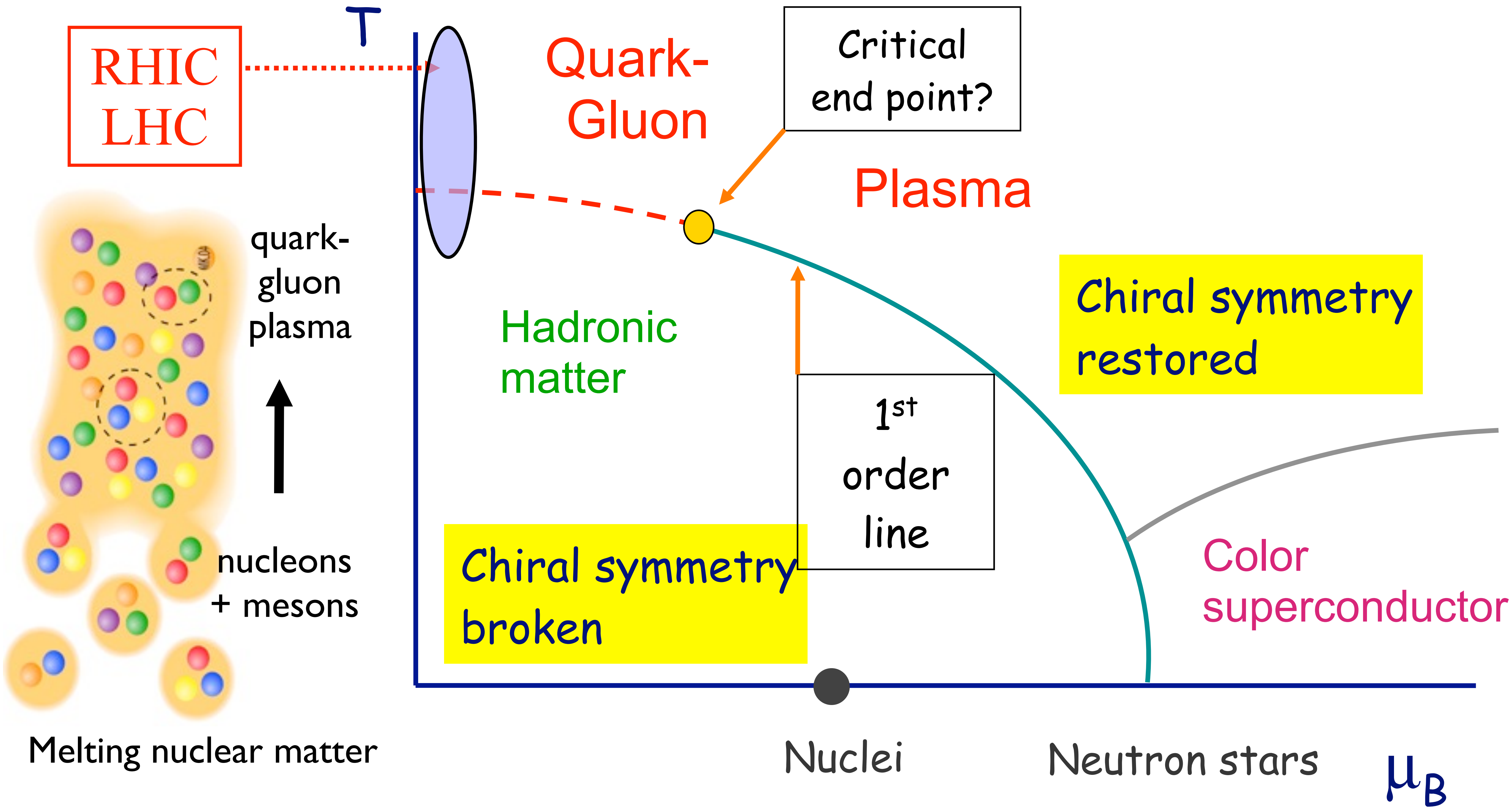

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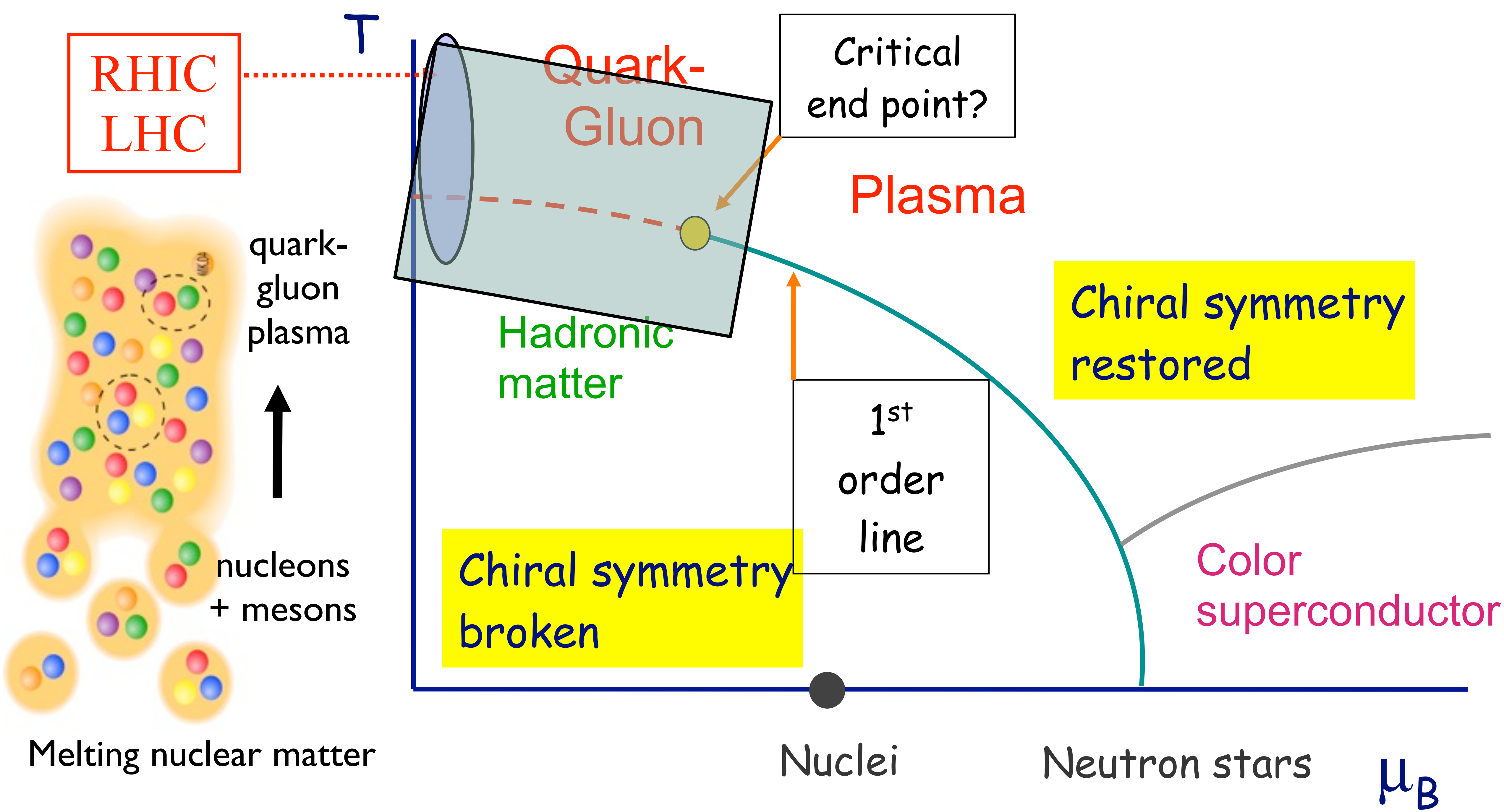


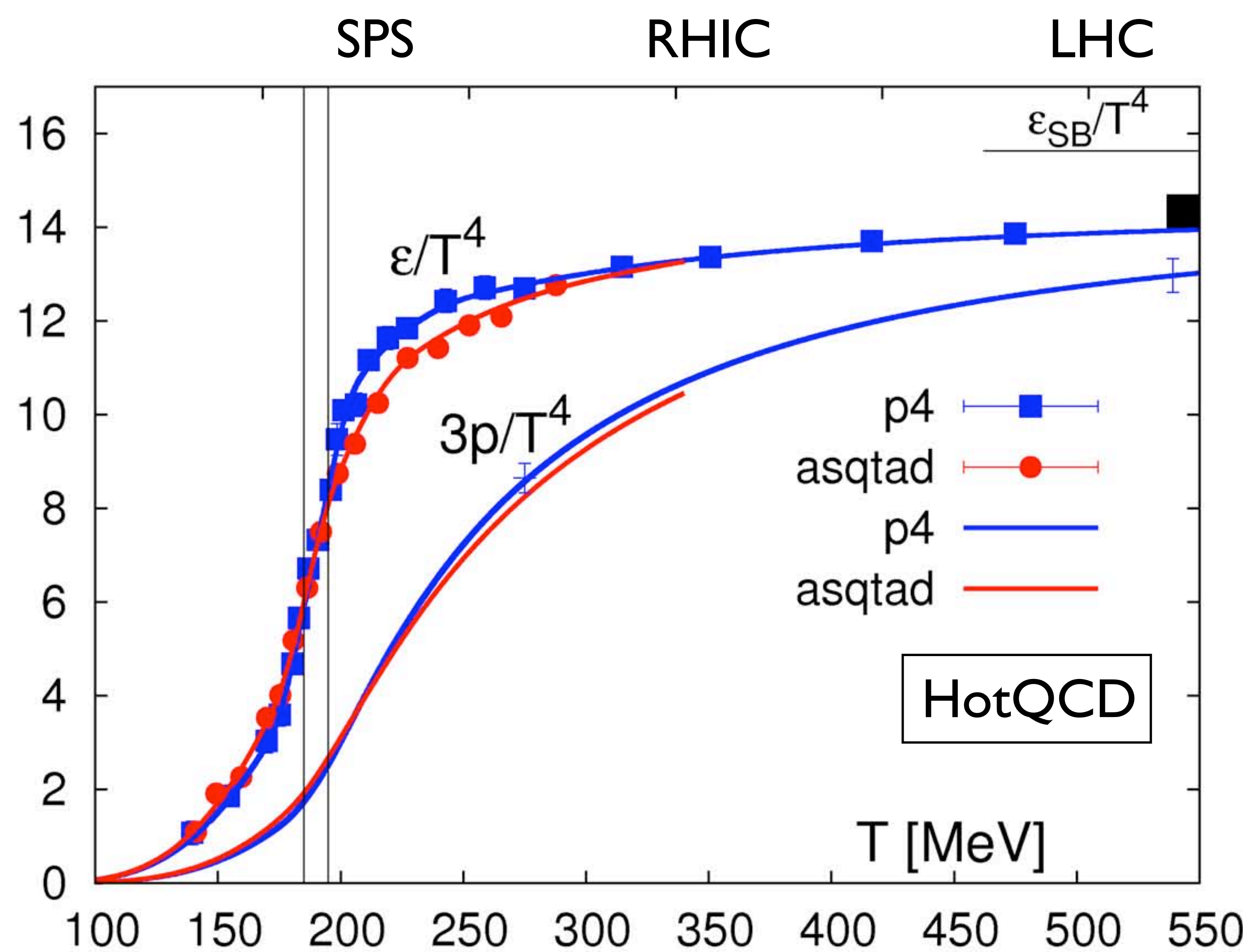
LQCD equation of state

gluons quarks

$$\text{Degrees of freedom}: \quad \nu = \left[(2\times 8) + \tfrac{7}{4}\times(2\times 3\times N_f)\right]\times\left(1 - O(g^2)\right)$$

spin | color | spin | color | flavor

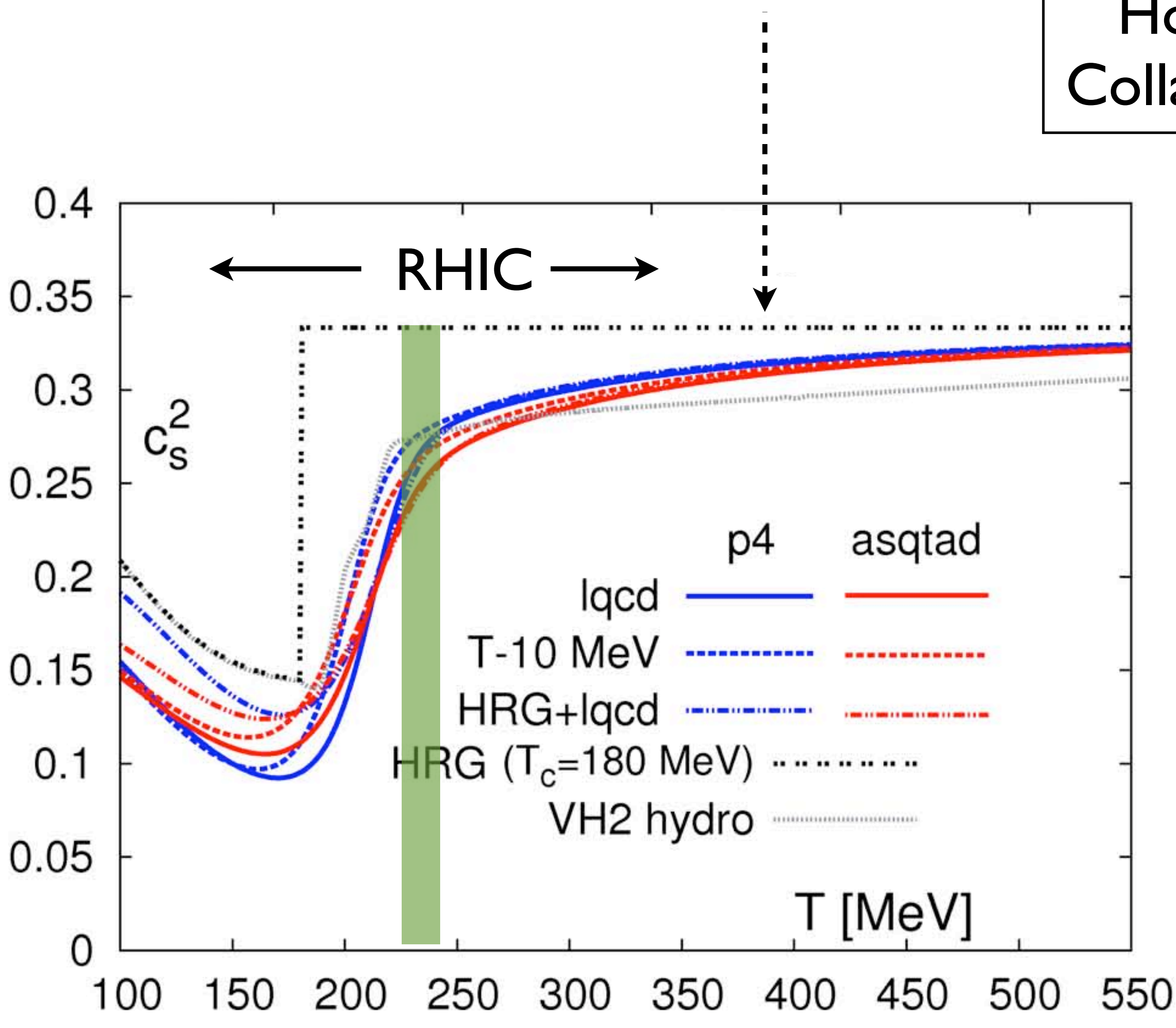
$$\frac{\varepsilon}{T^4} = \frac{\pi^2}{30}\nu_{\text{eff}}$$



QGP properties

Speed of sound

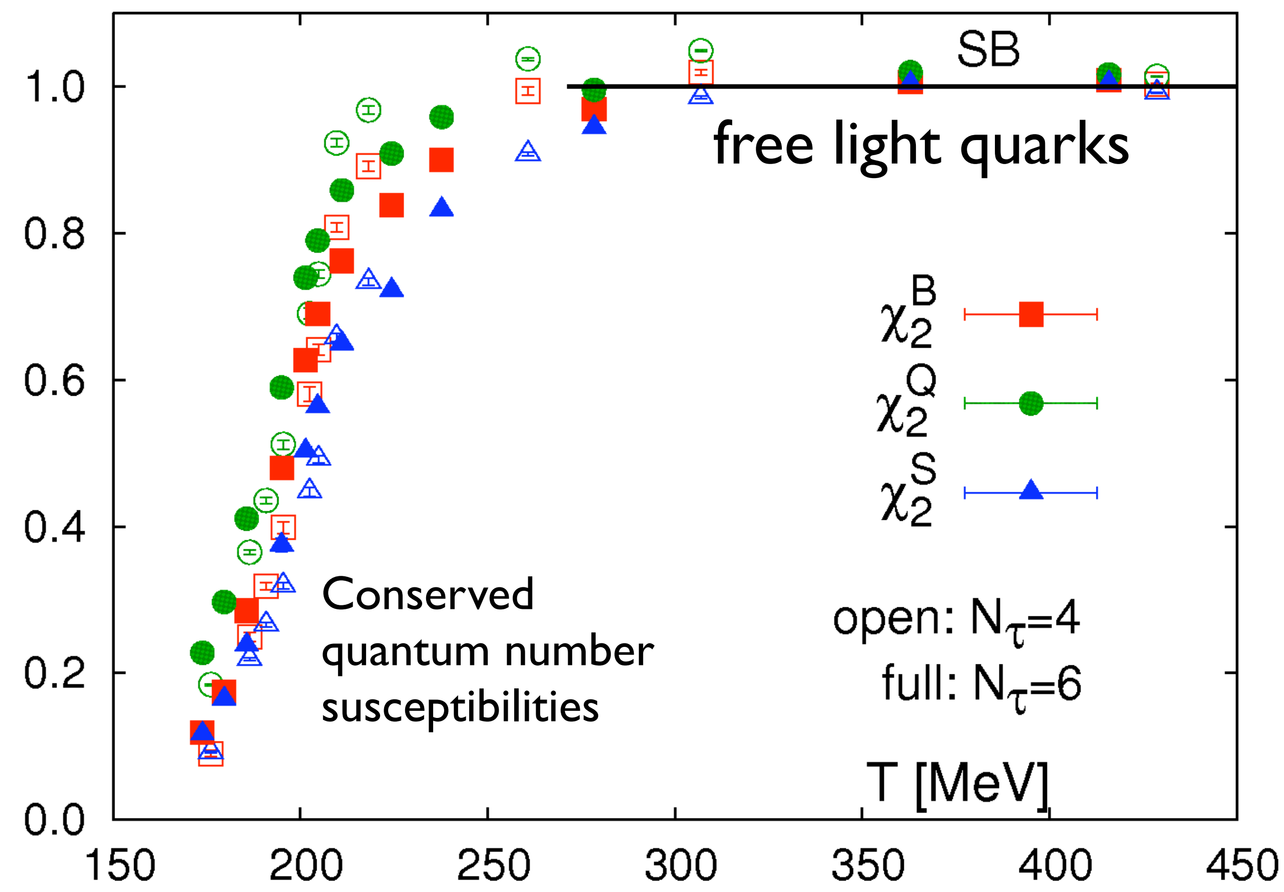
$$c_s^2 = \frac{1}{3}$$



Susceptibilities χ

HotQCD Collaboration

B = baryon number
Q = electric charge
S = strangeness



Observables

Which properties of hot QCD matter can we hope to determine from relativistic heavy ion data ?

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$$\left.\begin{aligned} \hat{q} &= \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F^{a+i}(y^-) F_i^{a+}(0) \rangle \\ \hat{e} &= \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle i\partial^- A^{a+}(y^-) A^{a+}(0) \rangle \\ \hat{e}_2 &= \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F^{a+-}(y^-) F^{a+-}(0) \rangle \end{aligned}\right\}$$

Momentum/energy diffusion:
parton energy loss
modified jet fragmentation

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Color screening: Quarkonium states

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Easy for LQCD

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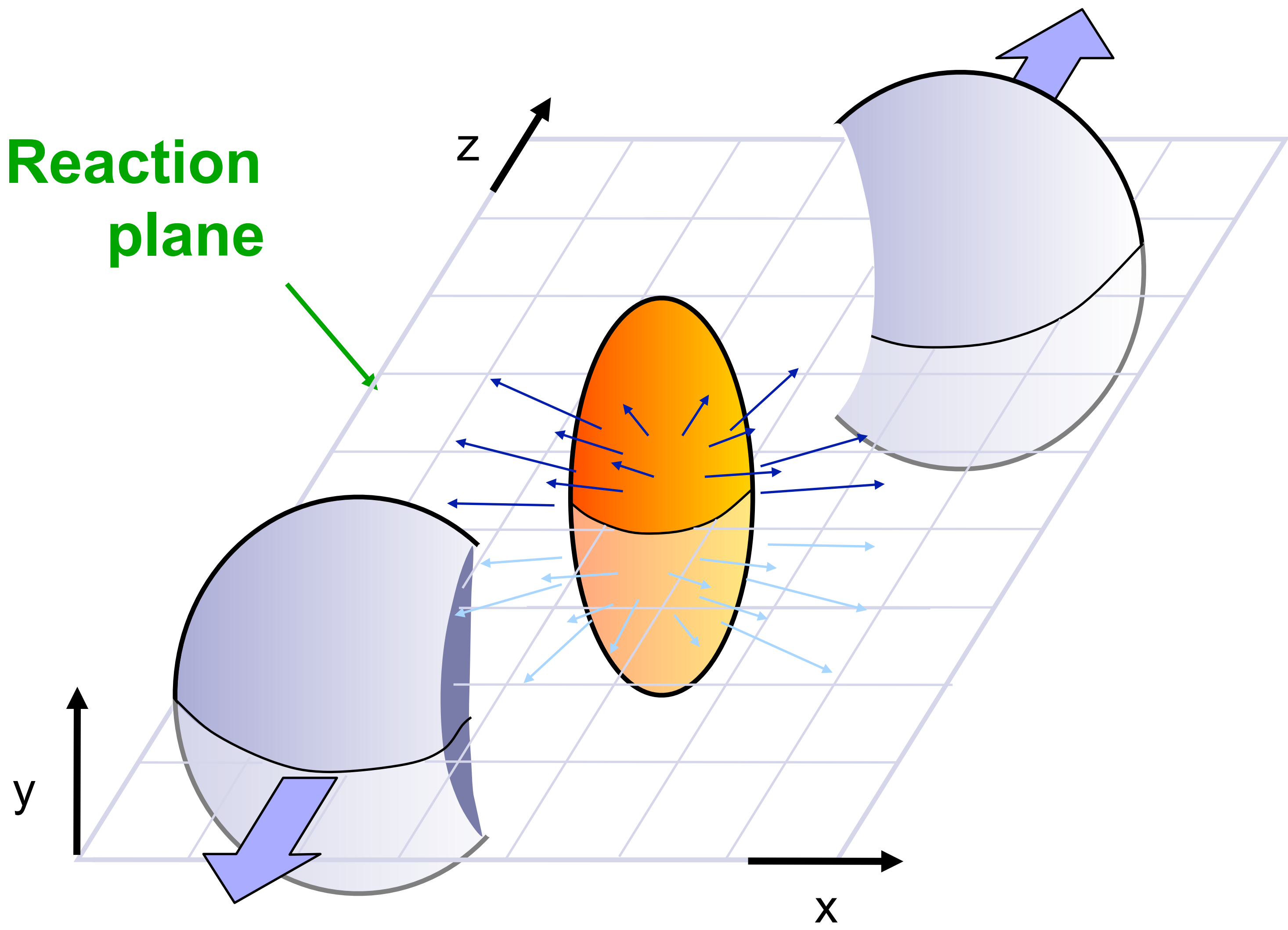
Color screening: Quarkonium states

Part 2

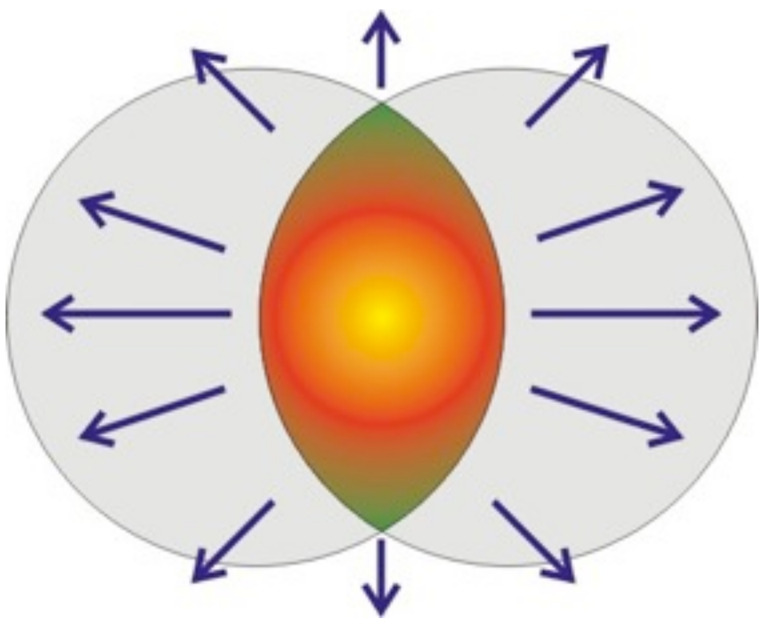
The (almost)

"Perfect Liquid"

Elliptic Flow (v_2)



v_2 = cos(2ϕ)
coefficient of the
azimuthal distribution

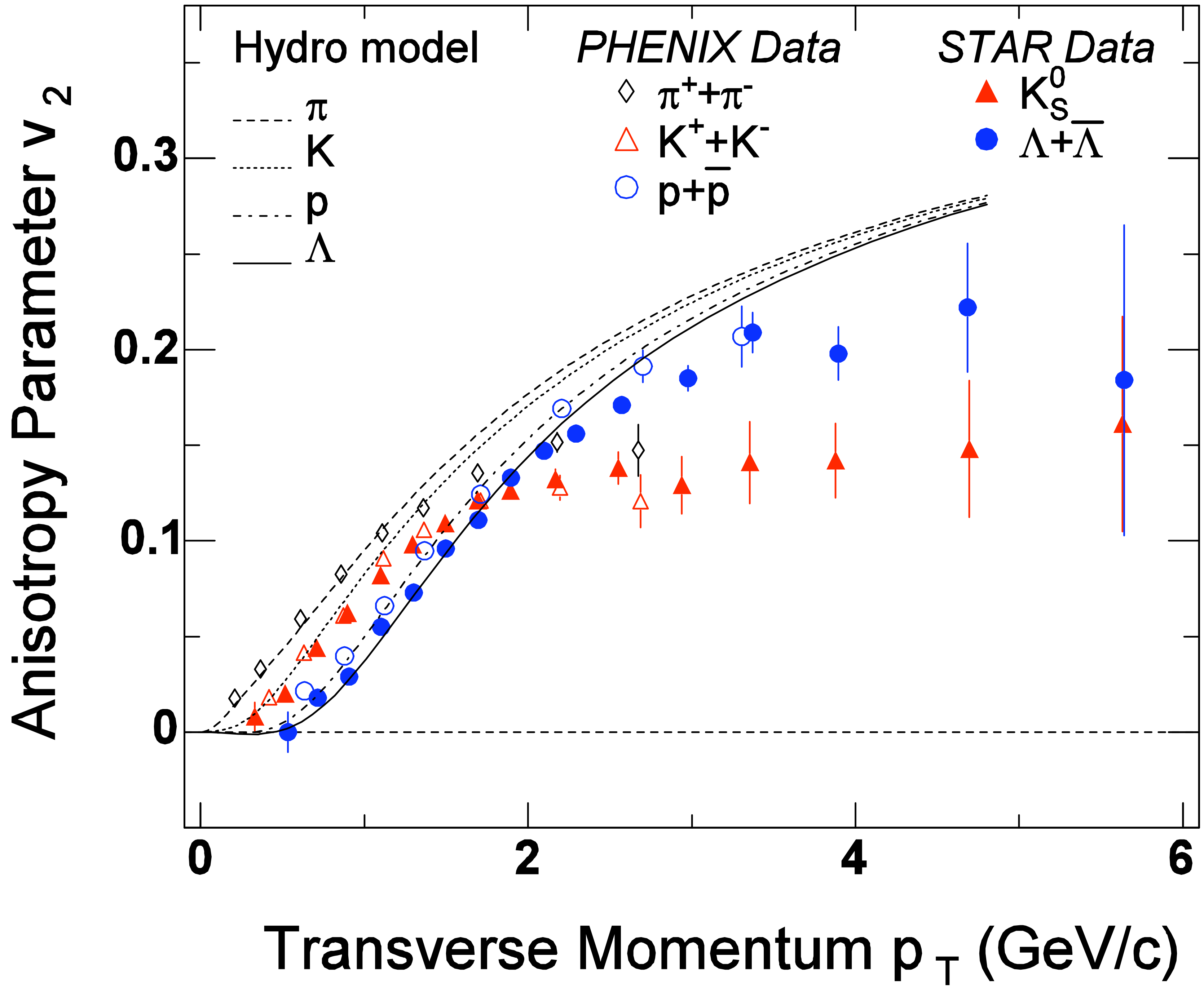


Hydrodynamics:

Flow is generated by ∇P

∇P(↔) > ∇P(↕)

$v_2(p_T)$ vs. hydrodynamics



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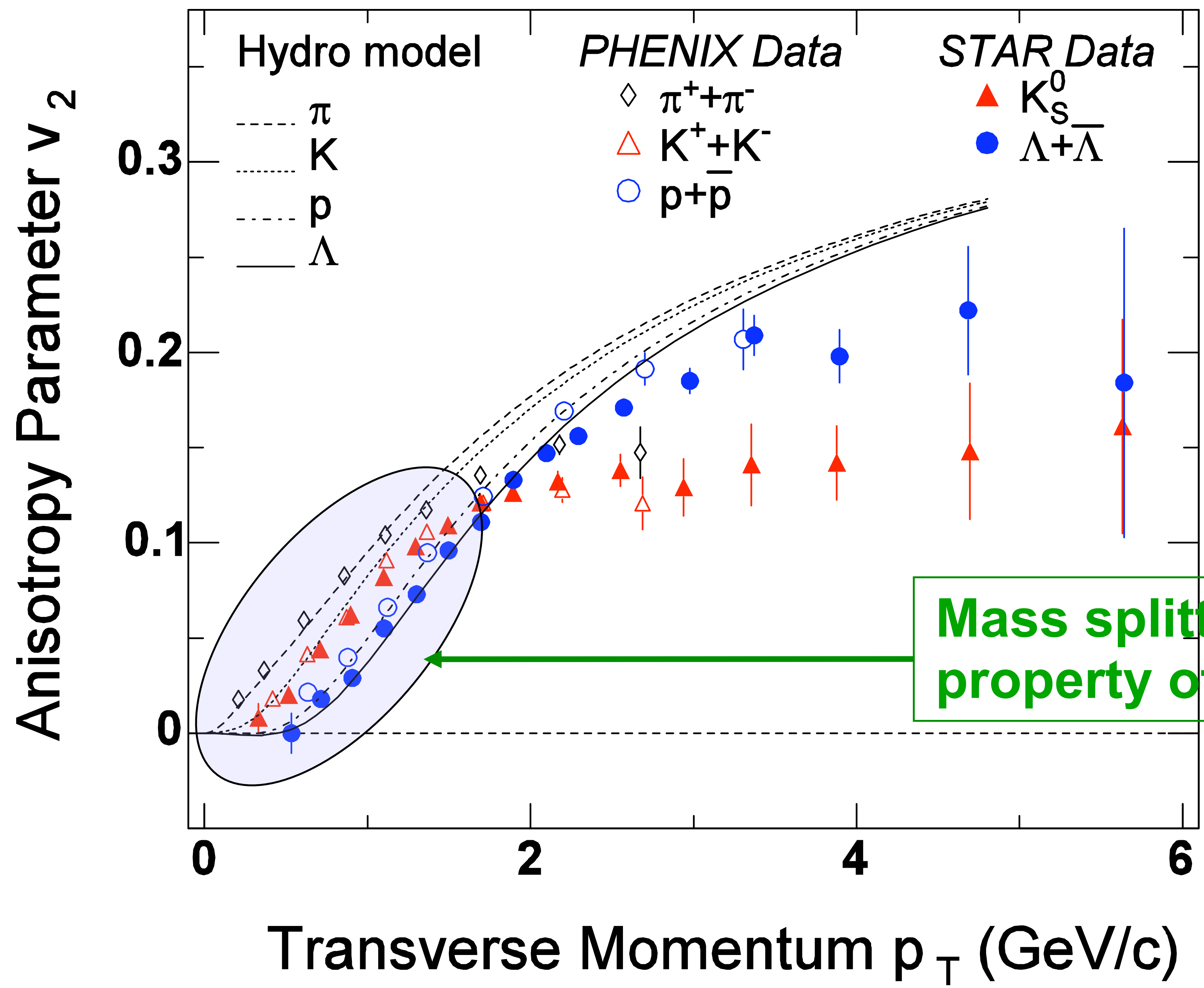
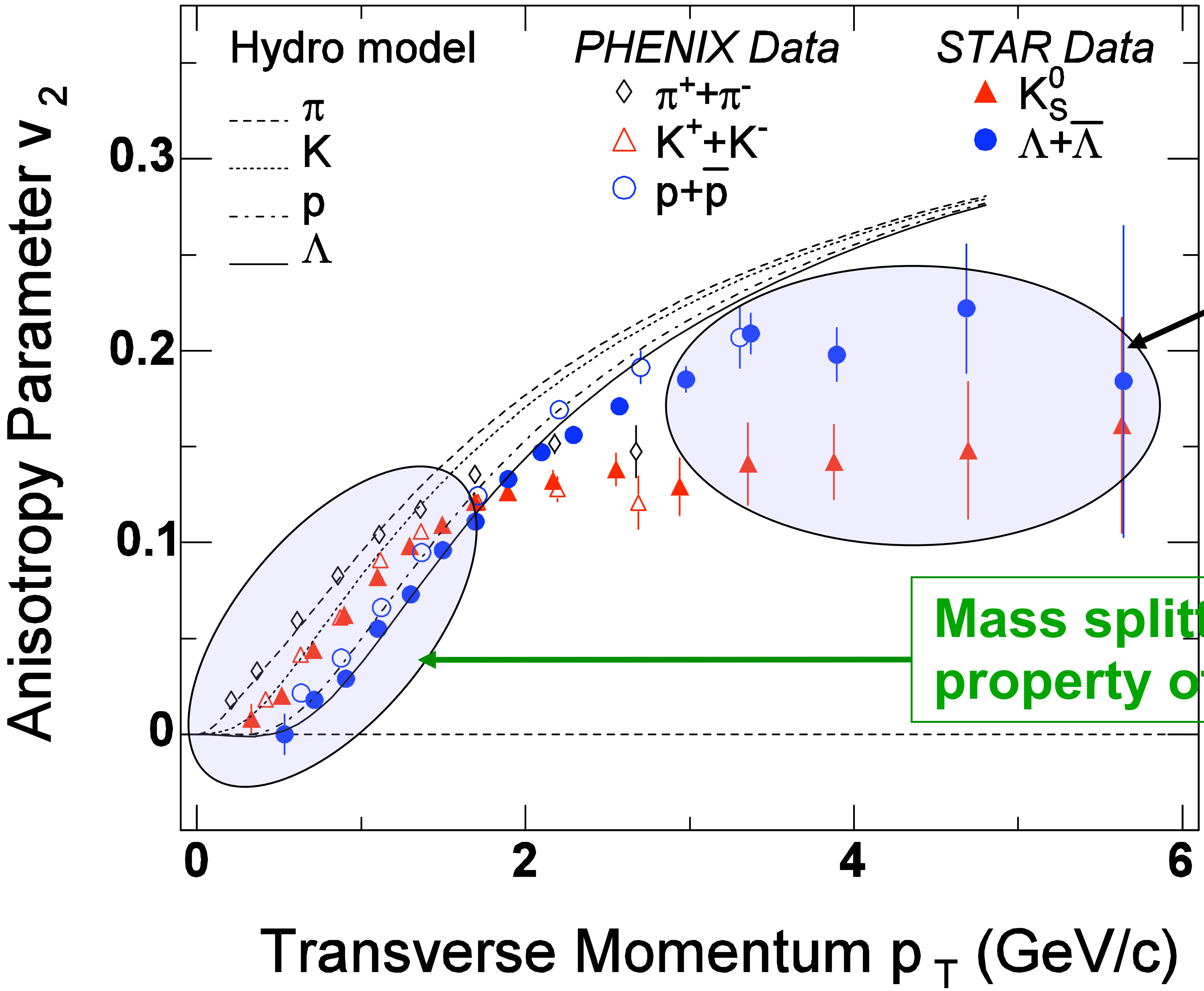


Mass splitting characteristic property of hydrodynamics

$v_2(p_T)$ vs. hydrodynamics



Failure of ideal hydrodynamics tells us how hadrons form

Mass splitting characteristic property of hydrodynamics

Elliptic flow "measures" η_{QGP}

We finally have a complete, causal formulation of relativistic viscous hydrodynamics:

Shear viscosity

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with} \quad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \Pi^{\mu\nu}$$

$$\tau_\Pi \left[\frac{d\Pi^{\mu\nu}}{d\tau} + \left(u^\mu \Pi^{\nu\lambda} + u^\nu \Pi^{\mu\lambda} \right) \frac{du^\lambda}{d\tau} \right] = \eta \left(\partial^\mu u^\nu + \partial^\nu u^\mu - \text{trace} \right) - \Pi^{\mu\nu}$$

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$$\begin{aligned}
\Pi &= \Pi_{\rm NS} - \tau_\Pi \, \dot{\Pi} \\
&+ \tau_{\Pi q} \, q \cdot \dot{u} - \ell_{\Pi q} \, \partial \cdot q - \zeta \, \hat{\delta}_0 \, \Pi \, \theta \\
&+ \lambda_{\Pi q} \, q \cdot \nabla \alpha + \lambda_{\Pi\pi} \, \pi^{\mu\nu} \sigma_{\mu\nu} \, , \\
q^\mu &= q^\mu_{\rm NS} - \tau_q \, \Delta^{\mu\nu} \dot{q}_\nu \\
&- \tau_{q\Pi} \, \Pi \, \dot{u}^\mu - \tau_{q\pi} \, \pi^{\mu\nu} \dot{u}_\nu + \ell_{q\Pi} \nabla^\mu \Pi - \ell_{q\pi} \, \Delta^{\mu\nu} \partial^\lambda \pi_{\nu\lambda} + \tau_q \, \omega^{\mu\nu} q_\nu - \frac{\kappa}{\beta} \hat{\delta}_1 \, q^\mu \, \theta \\
&- \lambda_{qq} \, \sigma^{\mu\nu} q_\nu + \lambda_{q\Pi} \, \Pi \nabla^\mu \alpha + \lambda_{q\pi} \, \pi^{\mu\nu} \nabla_\nu \alpha \, , \\
\pi^{\mu\nu} &= \pi^{\mu\nu}_{\rm NS} - \tau_\pi \, \dot{\pi}^{<\mu\nu>} \\
&+ 2 \, \tau_{\pi q} \, q^{<\mu} \dot{u}^{\nu>} + 2 \, \ell_{\pi q} \, \nabla^{<\mu} q^{\nu>} + 2 \, \tau_\pi \, \pi_\lambda{}^{<\mu} \omega^{\nu>\lambda} - 2 \, \eta \, \hat{\delta}_2 \, \pi^{\mu\nu} \, \theta \\
&- 2 \, \tau_\pi \, \pi_\lambda{}^{<\mu} \sigma^{\nu>\lambda} - 2 \, \lambda_{\pi q} \, q^{<\mu} \nabla^{\nu>} \alpha + 2 \, \lambda_{\pi\Pi} \, \Pi \, \sigma^{\mu\nu} \, ,
\end{aligned}$$

Complete set of causal, dissipative relativistic hydrodynamics eqs.
(B. Betz & D. Rischke, JPG36, 2009)

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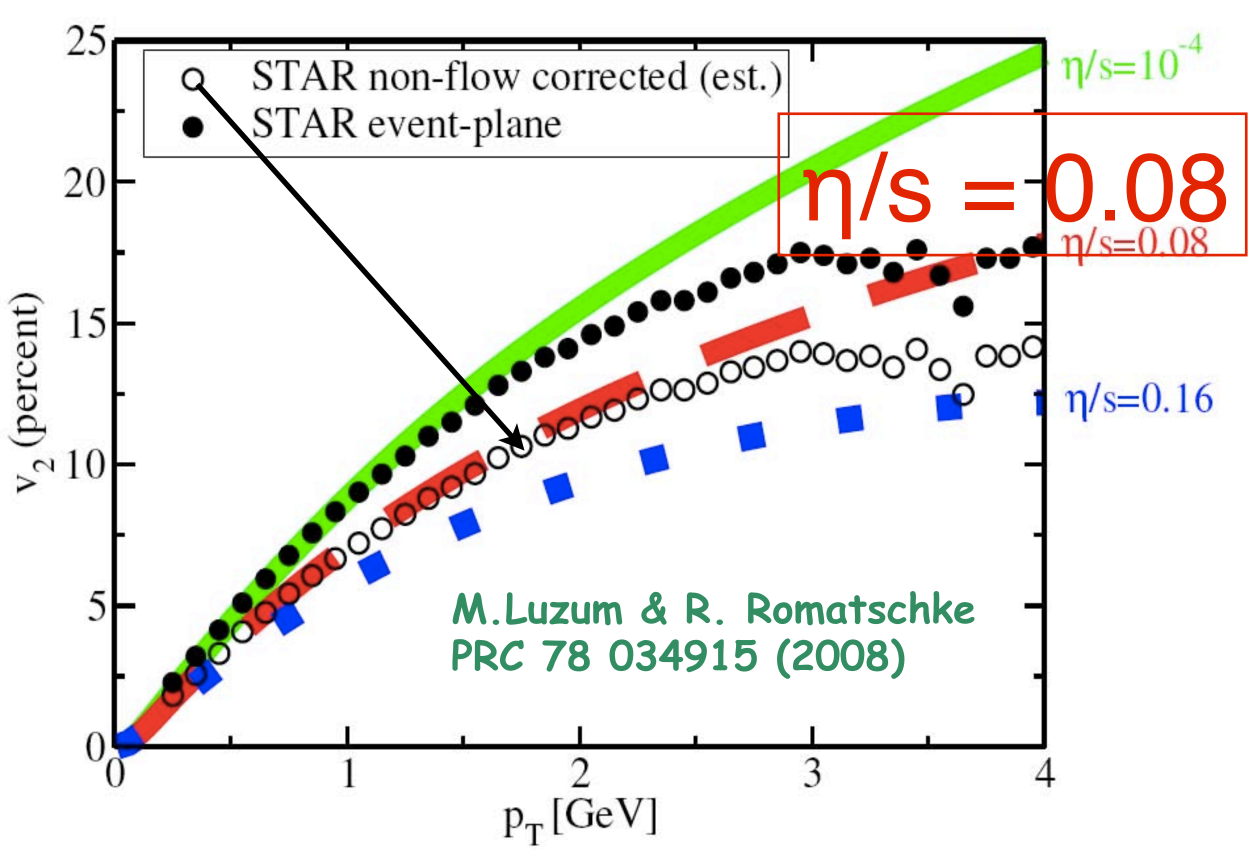
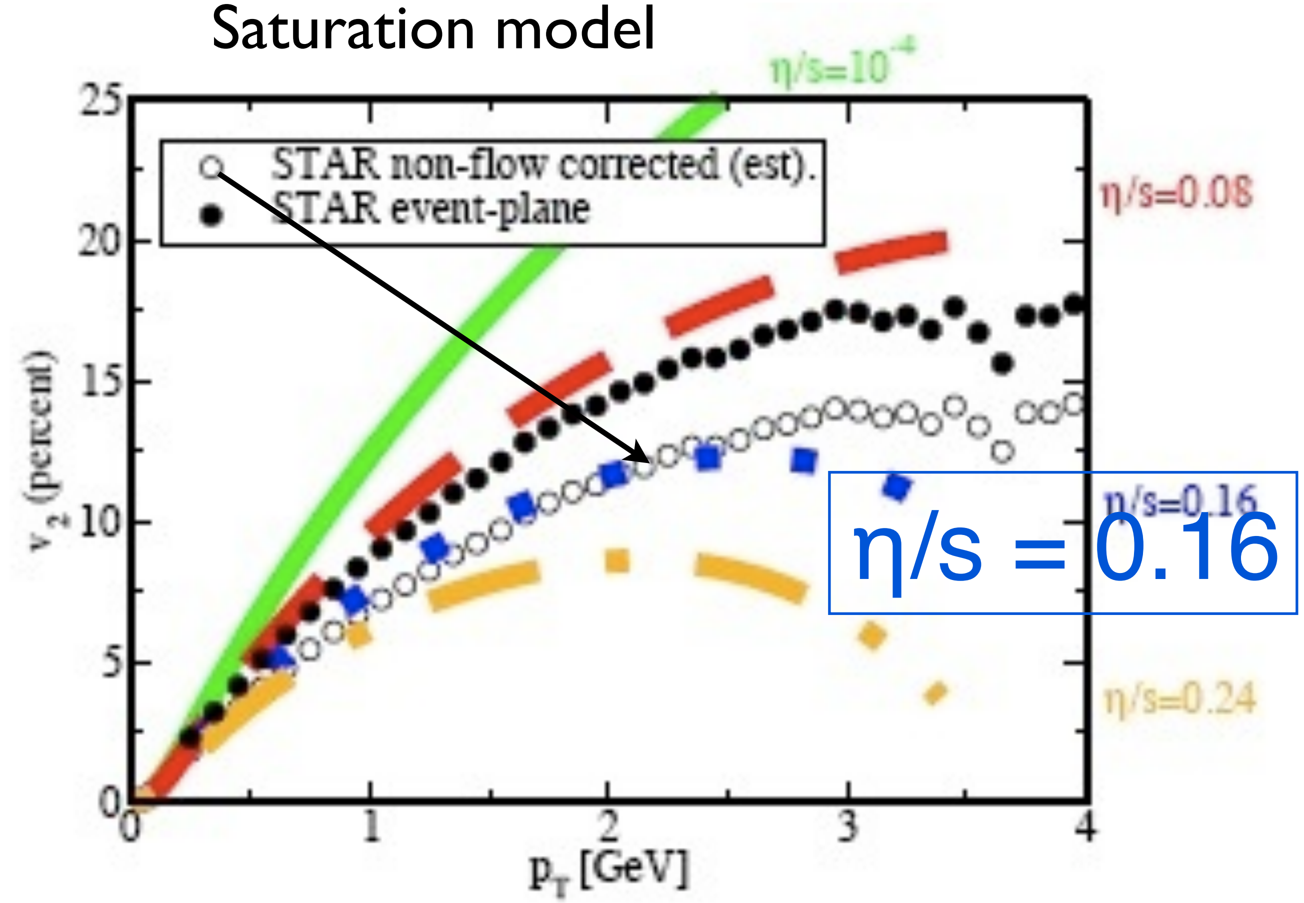
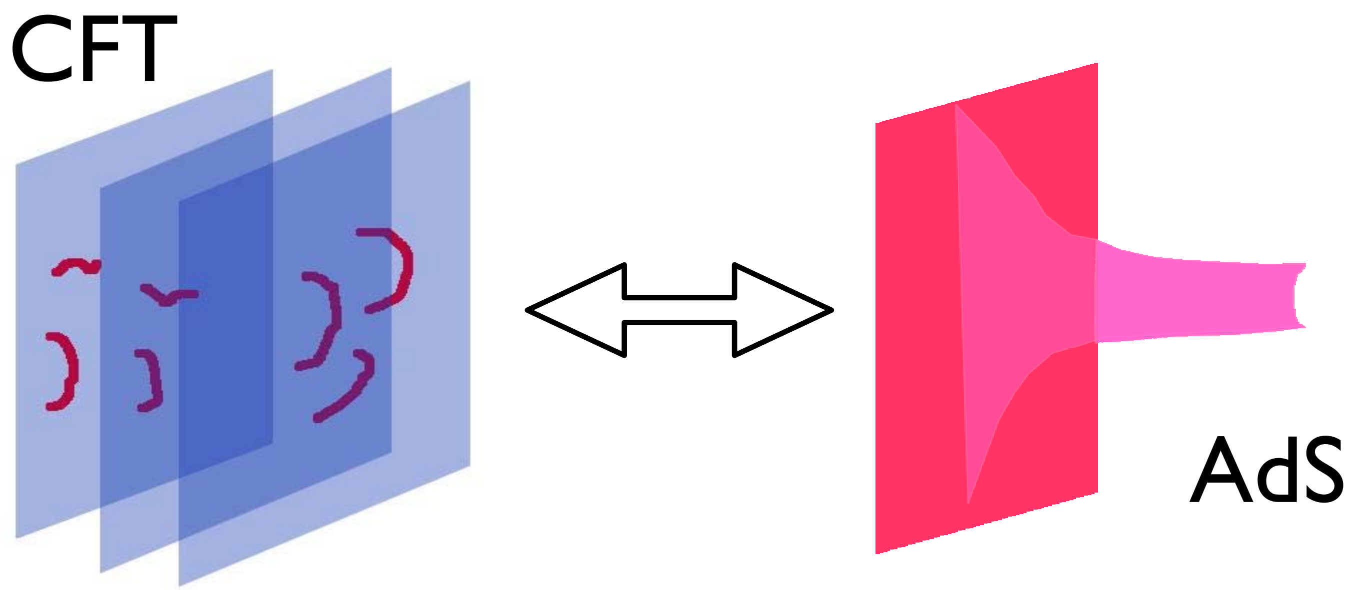
Glauber model



Saturation model

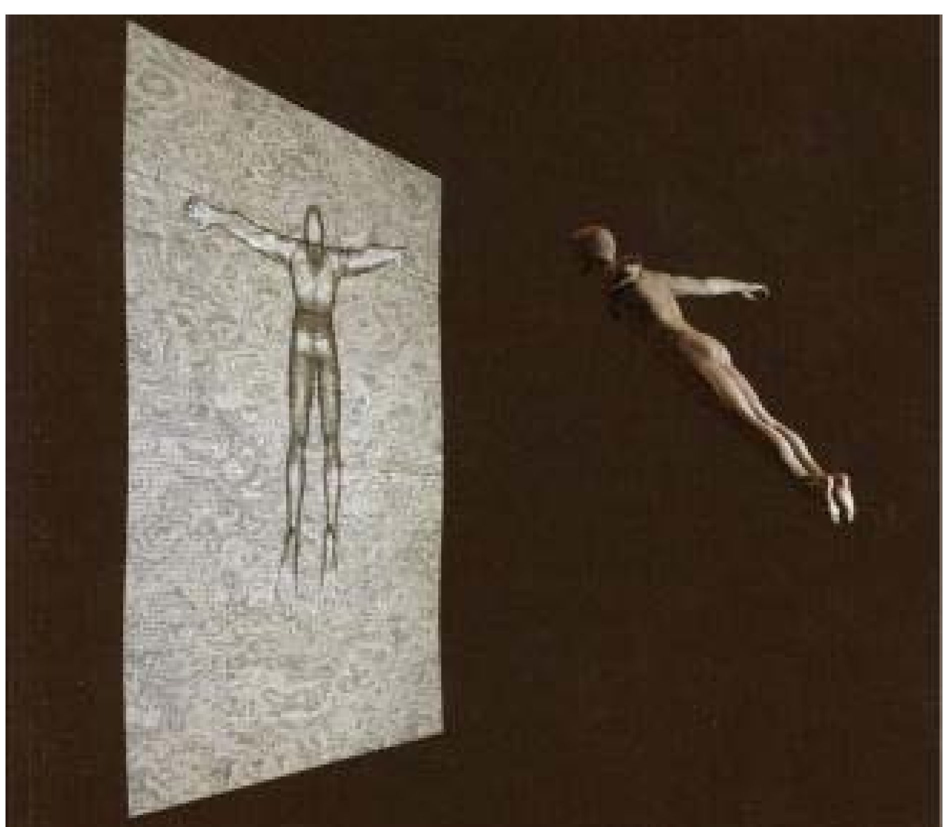


AdS/CFT duality



J. Maldacena (1997):

(3+1)-dim SYM theory in the N_c, $\sqrt{g^2 N_c} \to \infty$ limit is dual to classical super-gravity theory on AdS_5.



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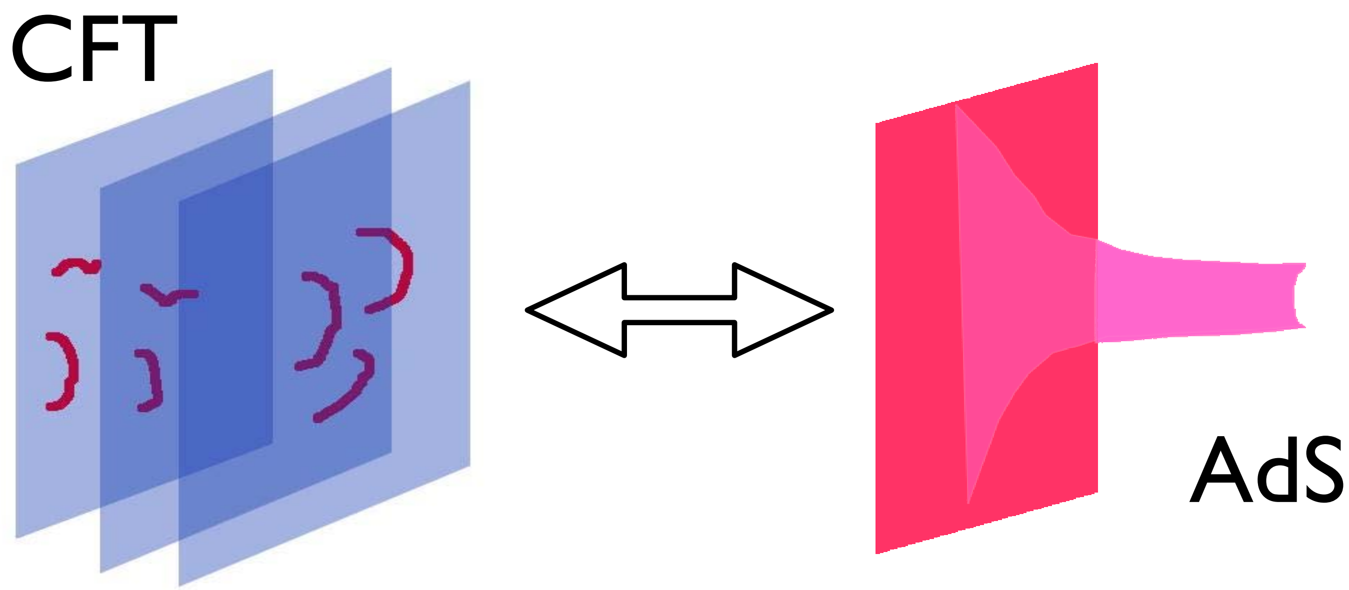

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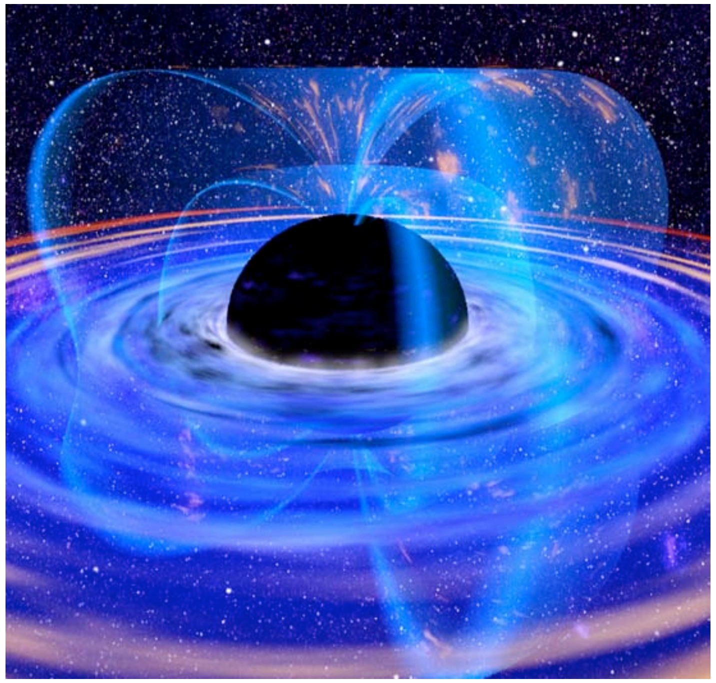
Application to RHIC invokes a 5th dim. BH.

Thermal CFT ↔ AdS BH Dictionary

Stress tensor ↔ Asymptotic metric

Entropy ↔ Horizon area

Viscosity ↔ Graviton absoption



Perfect fluid

Dissipation is dominated by absorption of gravitons on the black brane:

Universal bound ?

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

Kovtun, Son & Starinets (2005)

Similar bound in kinetic theory from unitarity limit of cross sections and/or uncertainty relation [Danielewicz & Gyulassy '85].

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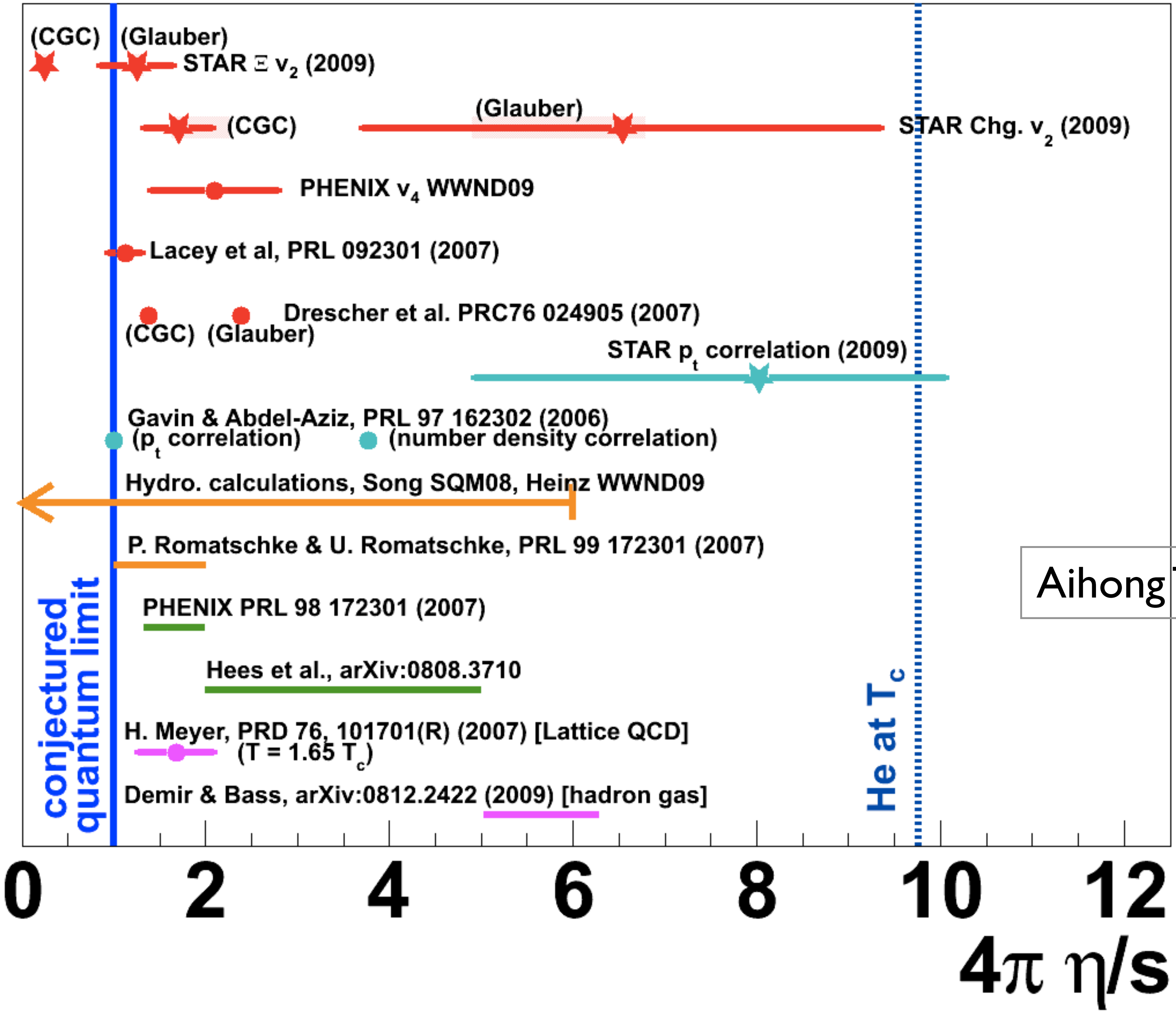
Bound is probably not completely universal, but far below η/s of any known material (except ultra-cold gases of fermionic atoms with unitary interactions)

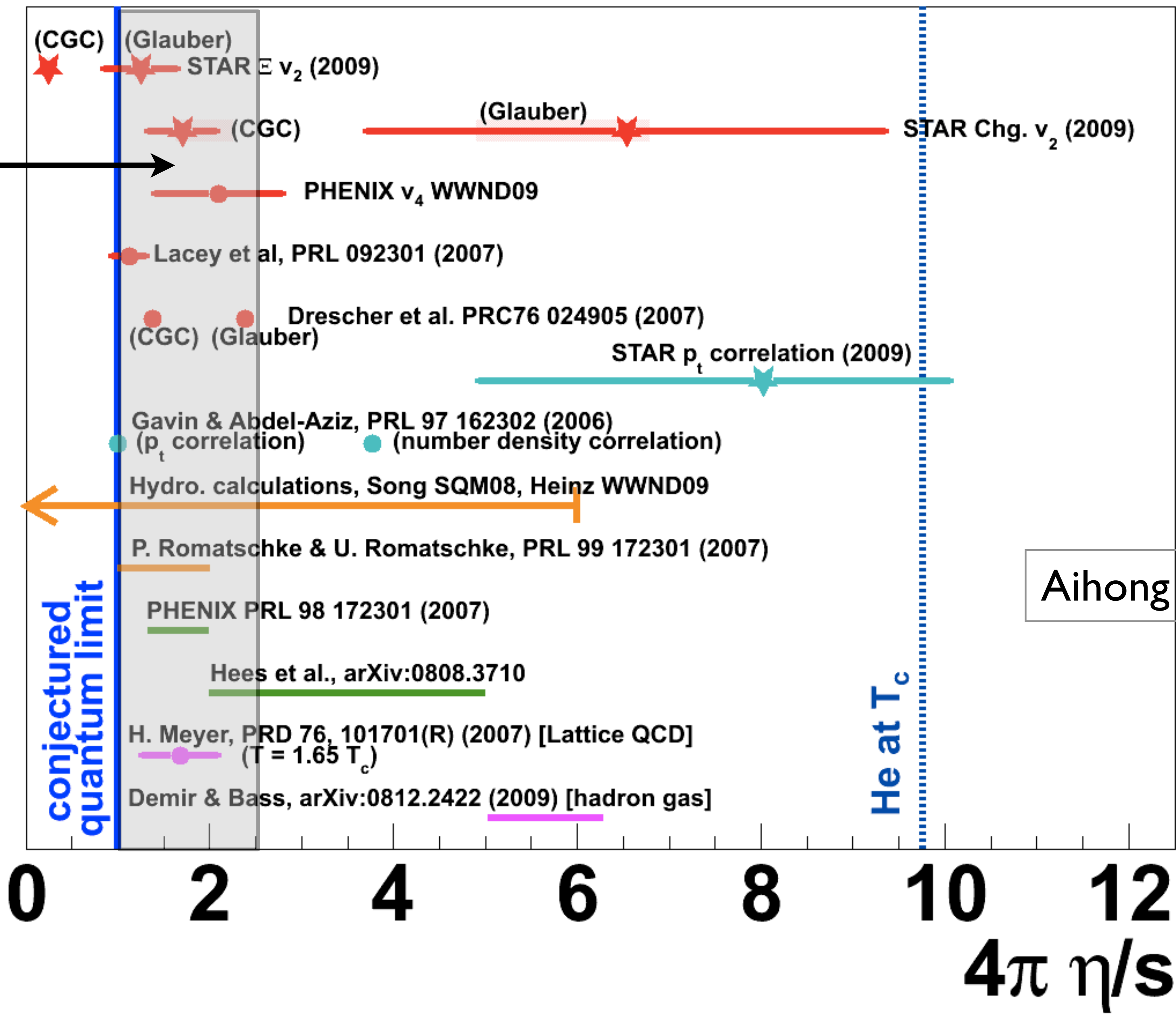
Hunting for perfection...



Aihong Tang (STAR)

Hunting for perfection...

My best bet



Aihong Tang (STAR)

Hunting for perfection...

Hunting for perfection...

We know how to settle the argument !

Viscous hydro
&
global data fit

Part 3

Collective Flow

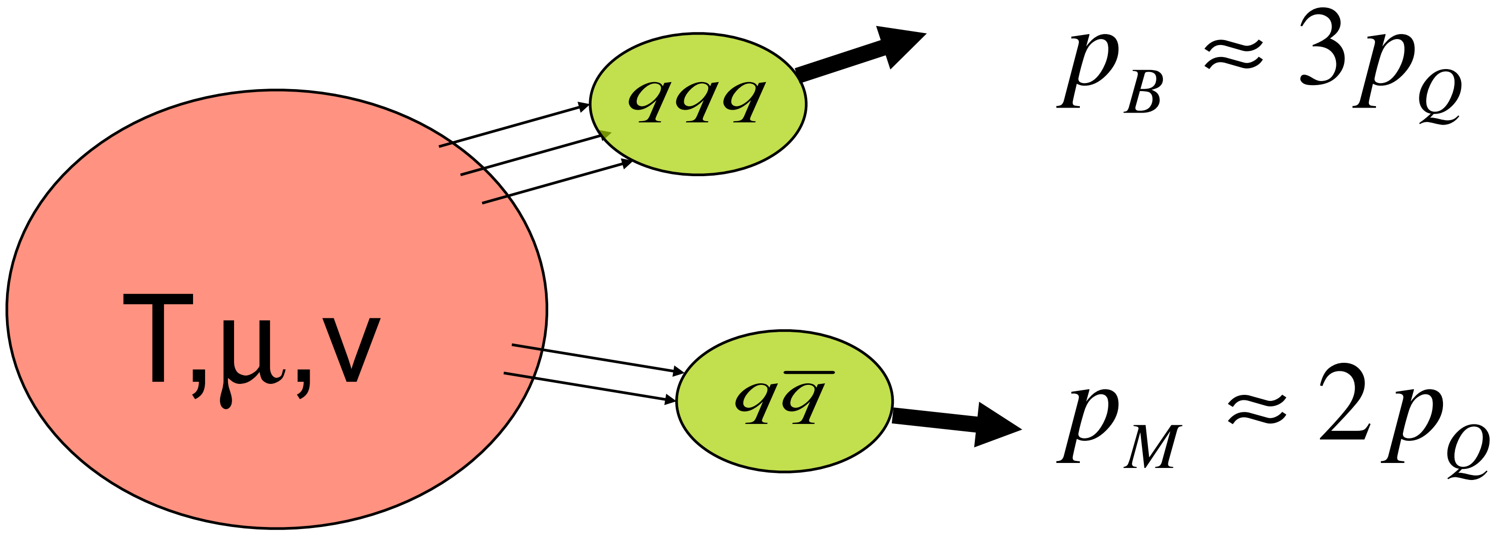
and

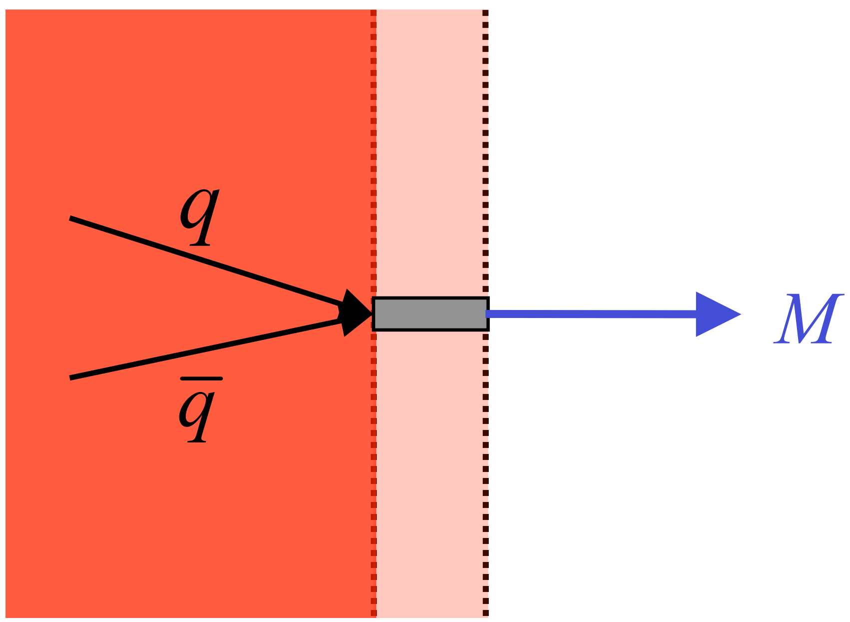
Deconfined Quarks

Bulk hadronization

Fast hadrons experience a rapid transition from medium to vacuum for fast hadrons

Sudden recombination





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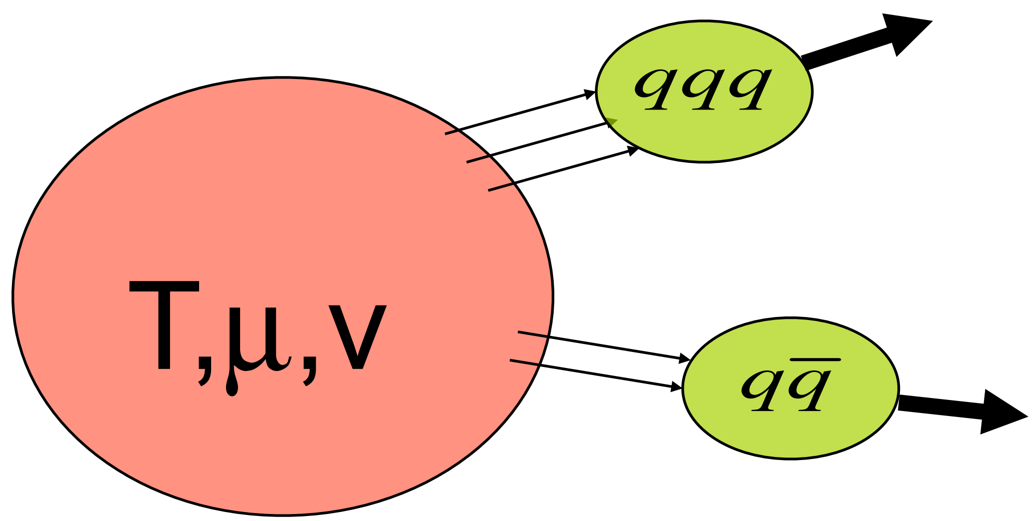
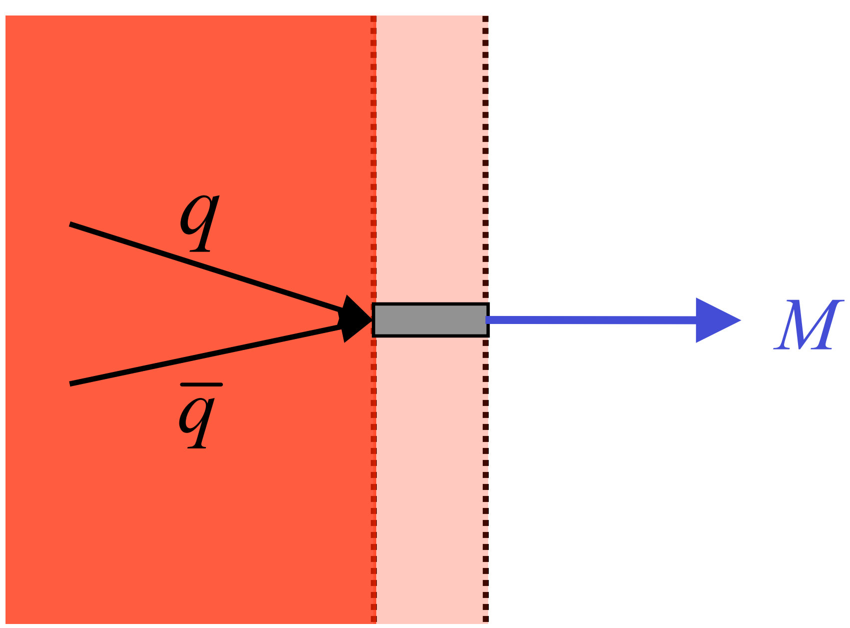


$$p_B \approx 3p_Q$$

$$p_M \approx 2p_Q$$

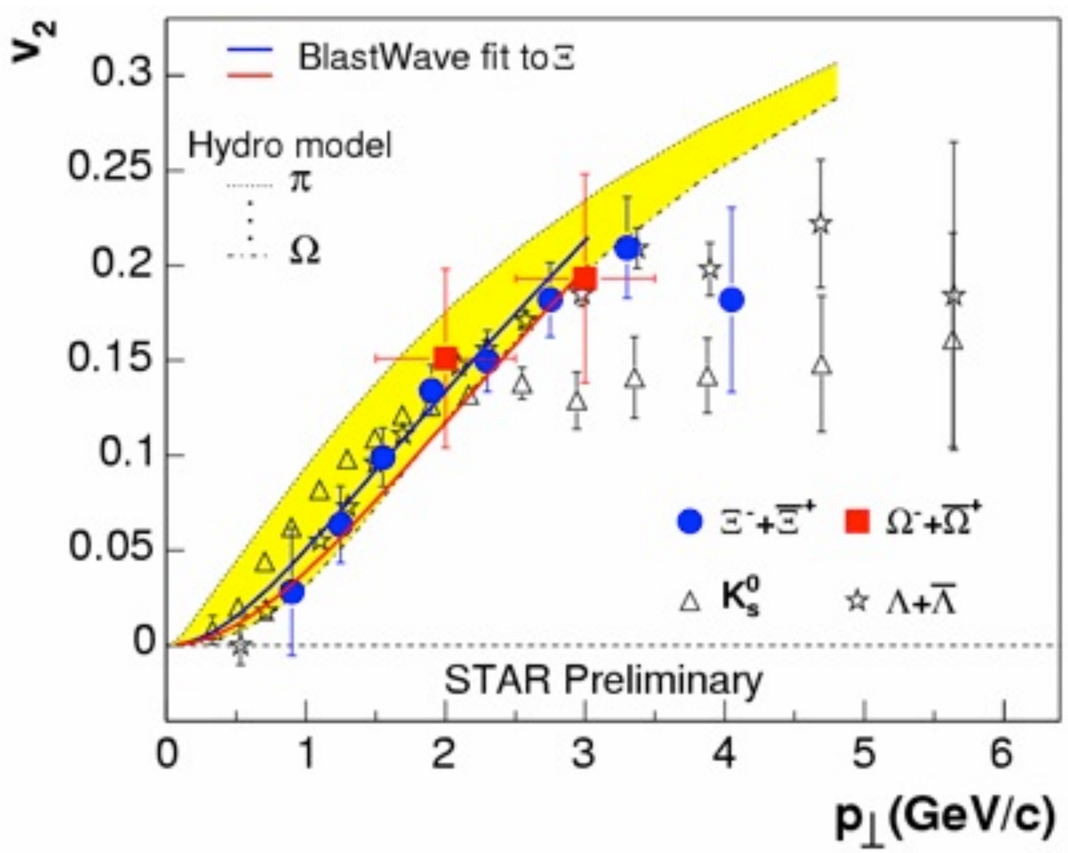


$$v_2^M(p_t) = 2v_2^Q\left(\frac{p_t}{2}\right)$$

$$v_2^B(p_t) = 3v_2^Q\left(\frac{p_t}{3}\right)$$

Quark number scaling of v_2

$$\frac{1}{2}v_2^M(p_t) = v_2^Q\left(\frac{p_t}{2}\right) \qquad \frac{1}{3}v_2^B(p_t) = v_2^Q\left(\frac{p_t}{3}\right)$$



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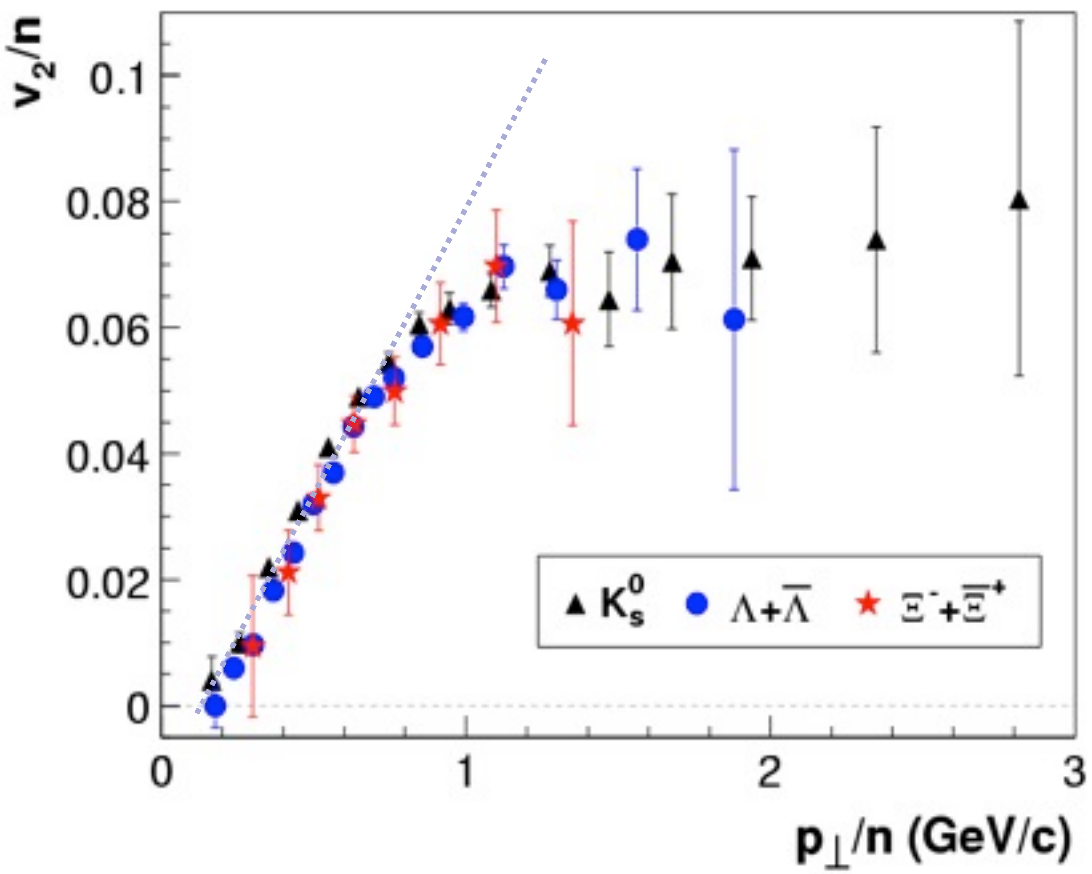

Emitting medium is composed of unconfined, flowing quarks.

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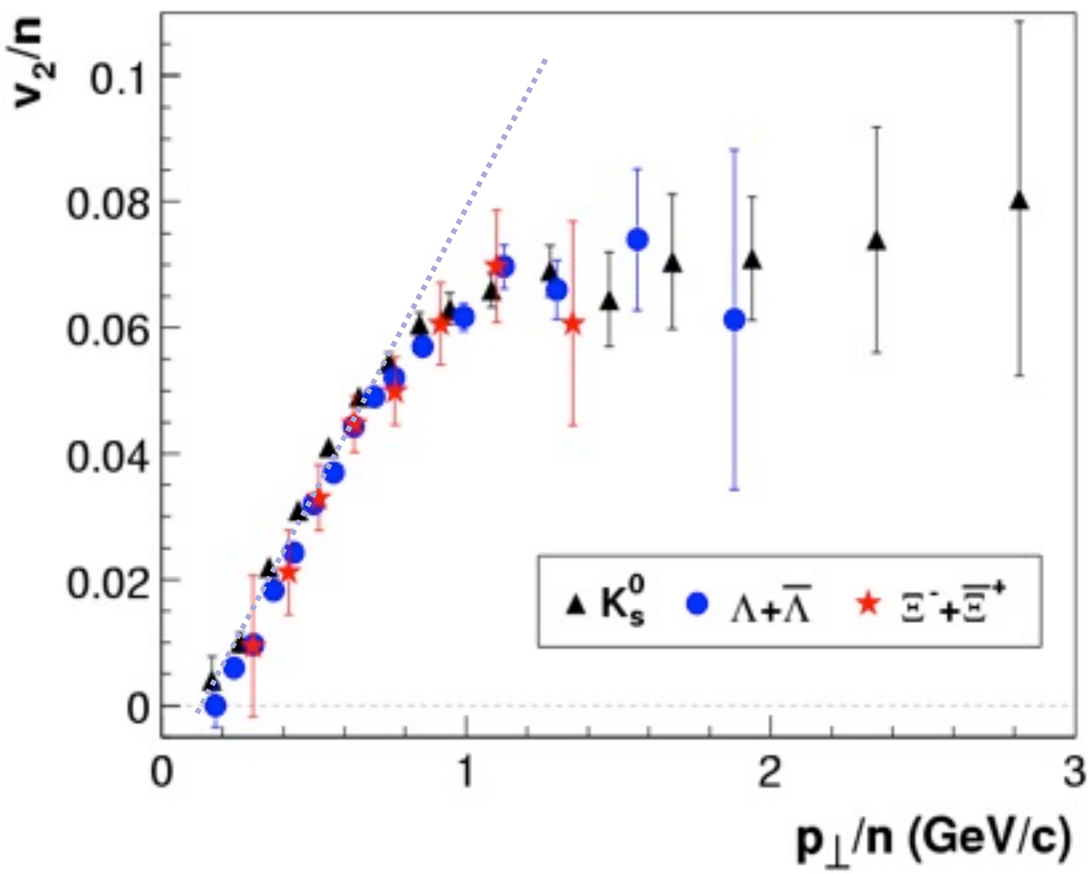
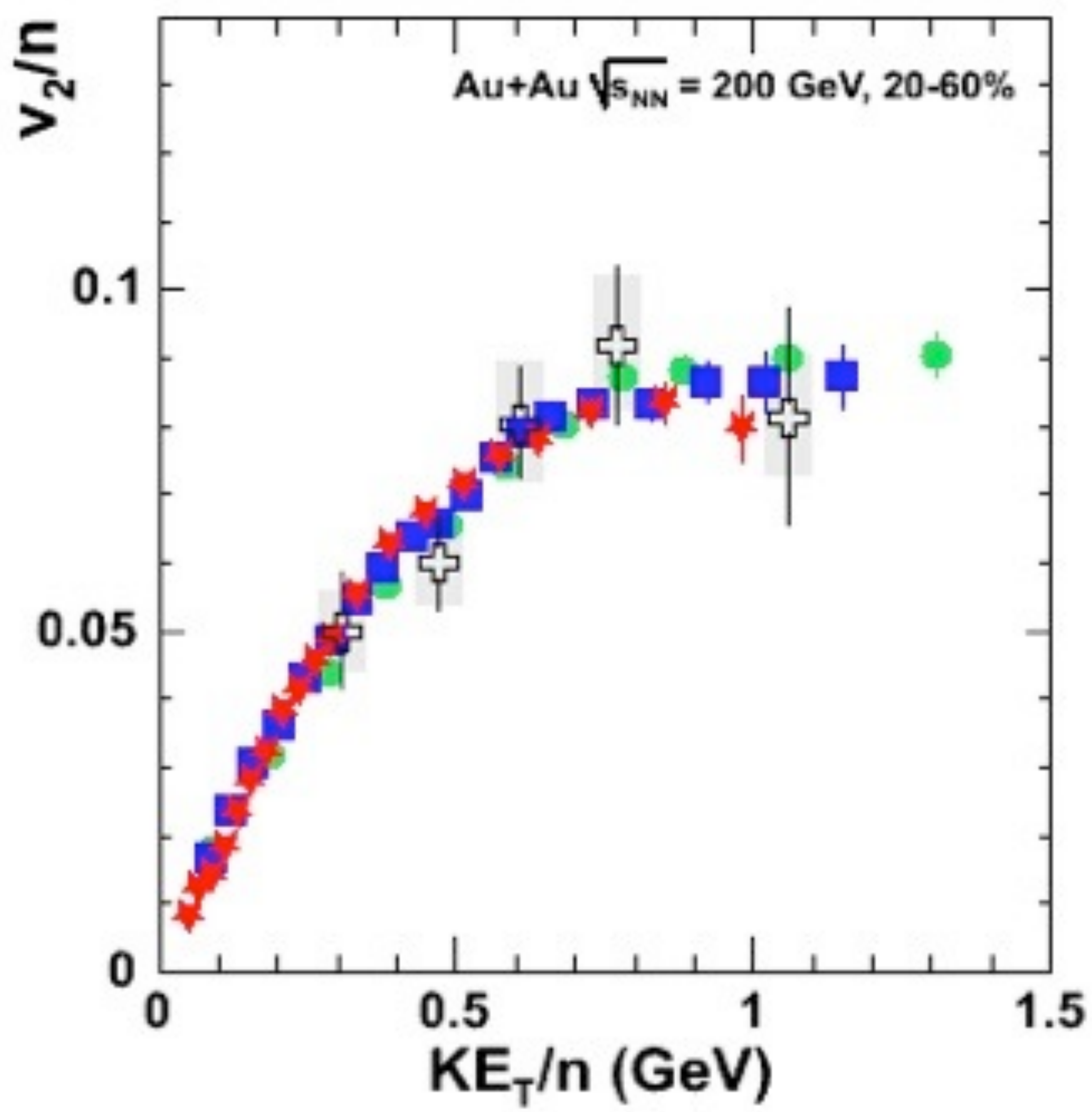
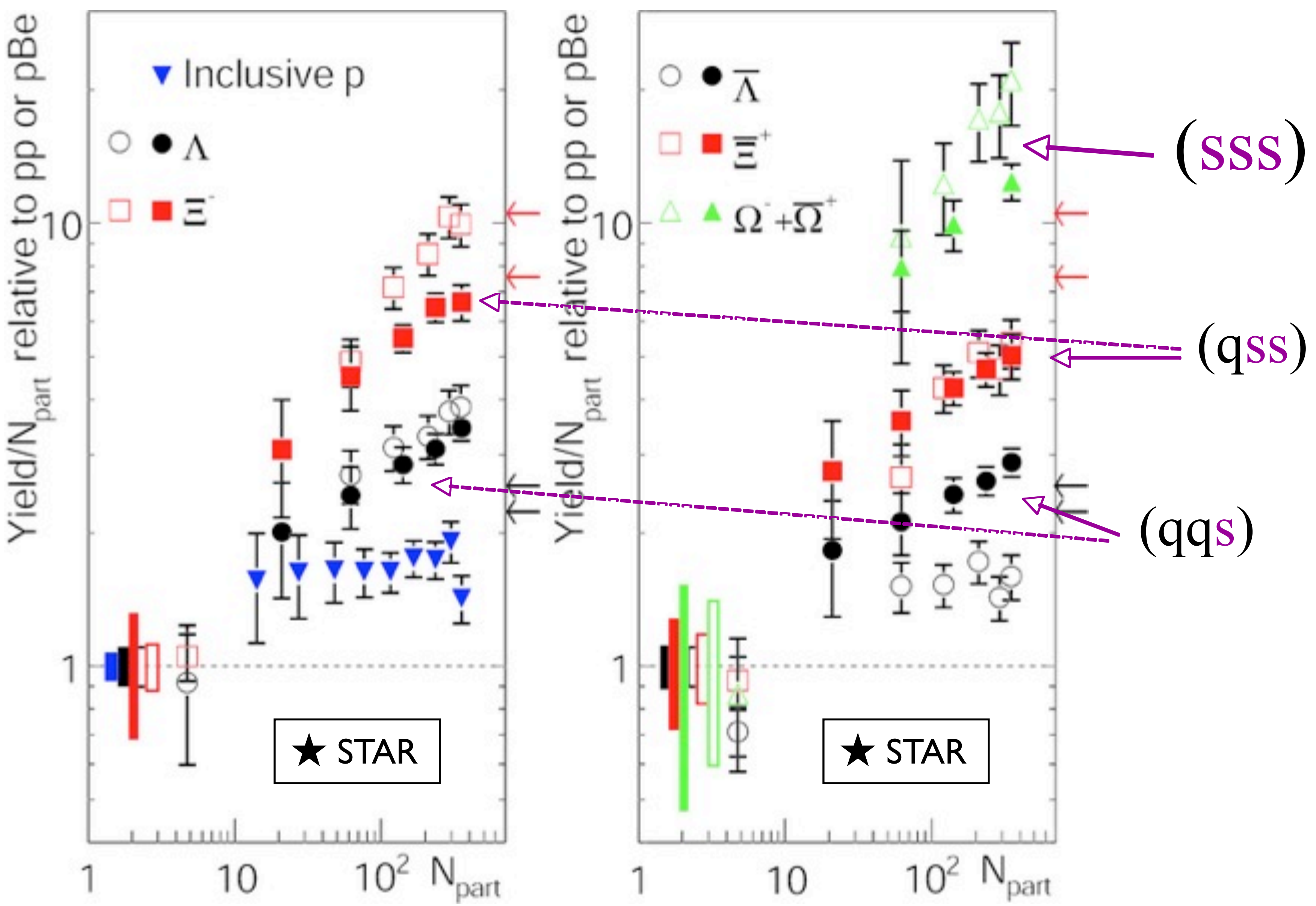




Emitting medium is composed of unconfined, flowing quarks.

The QGP is *strange* !

Strange hadron production is enhanced, as predicted,
Yield is chemically equilibrated with error of less than 4% !



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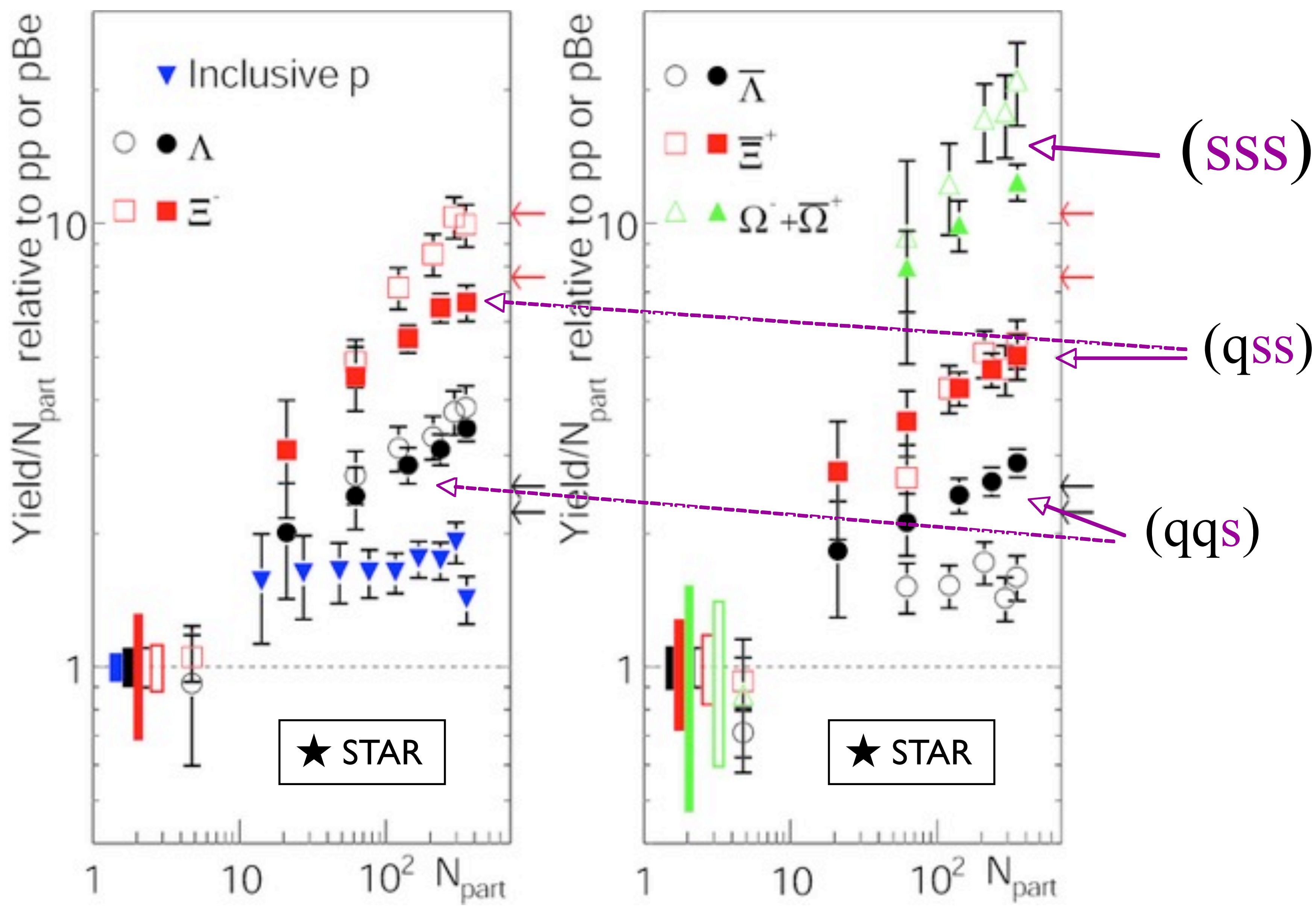


Combined with valence quark scaling law we can utilize strange quarks for ...

Quark spectroscopy

An amazing idea only experimentalists can come up with...

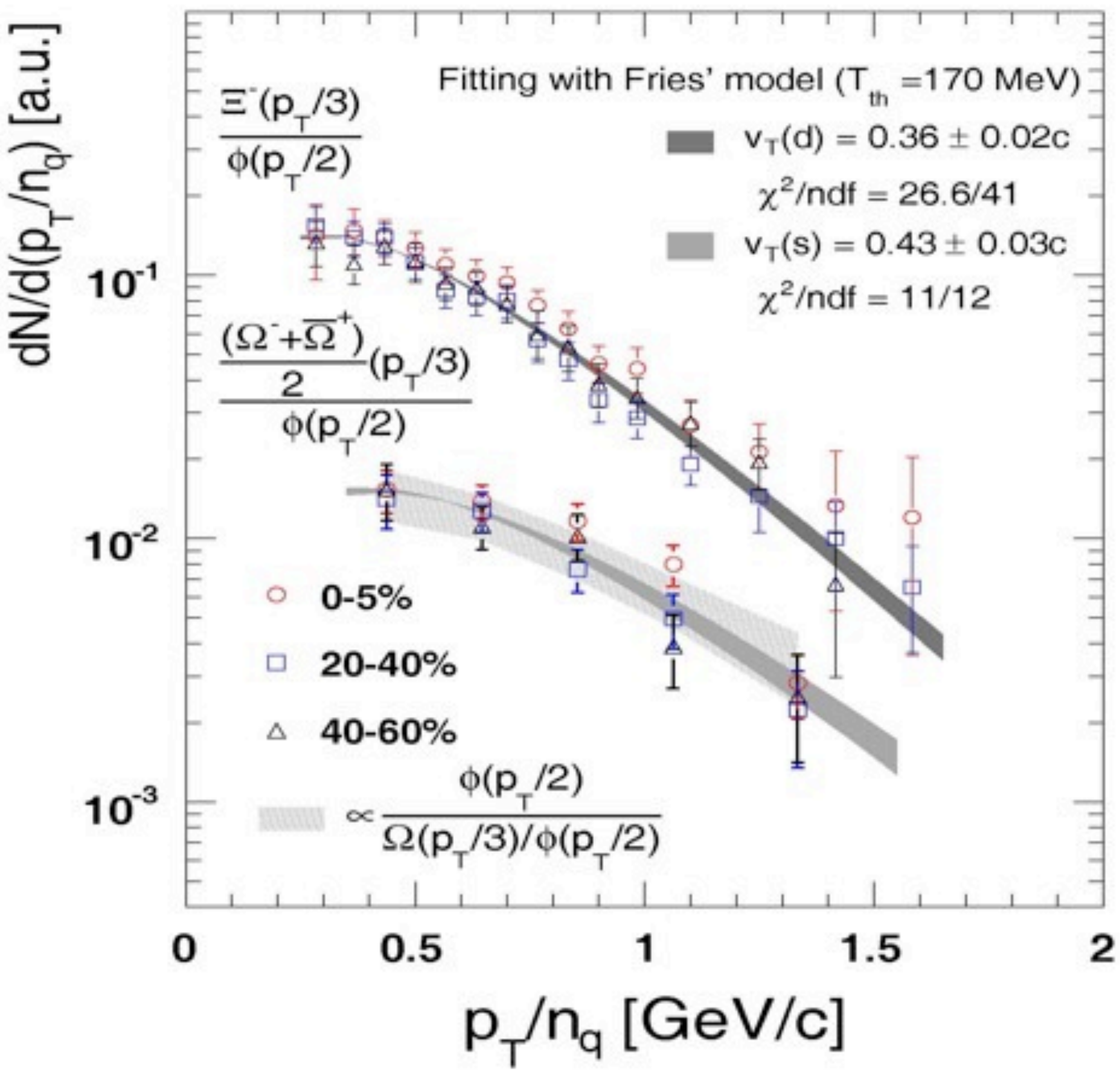
Use ratios of hadron spectra to infer valence quark spectra:

$$\Xi^- = (ssd), \quad \phi = (s\bar{s}), \quad \Omega^- = (sss)$$

$$d(p_T) = \frac{\Xi^-\left(\frac{1}{3}p_T\right)}{\phi\left(\frac{1}{2}p_T\right)}$$

$$s(p_T) = \frac{\Omega^-\left(\frac{1}{3}p_T\right)}{\phi\left(\frac{1}{2}p_T\right)} \propto \frac{\left[\phi\left(\frac{1}{2}p_T\right)\right]^2}{\Omega^-\left(\frac{1}{3}p_T\right)}$$

STAR/UCLA group



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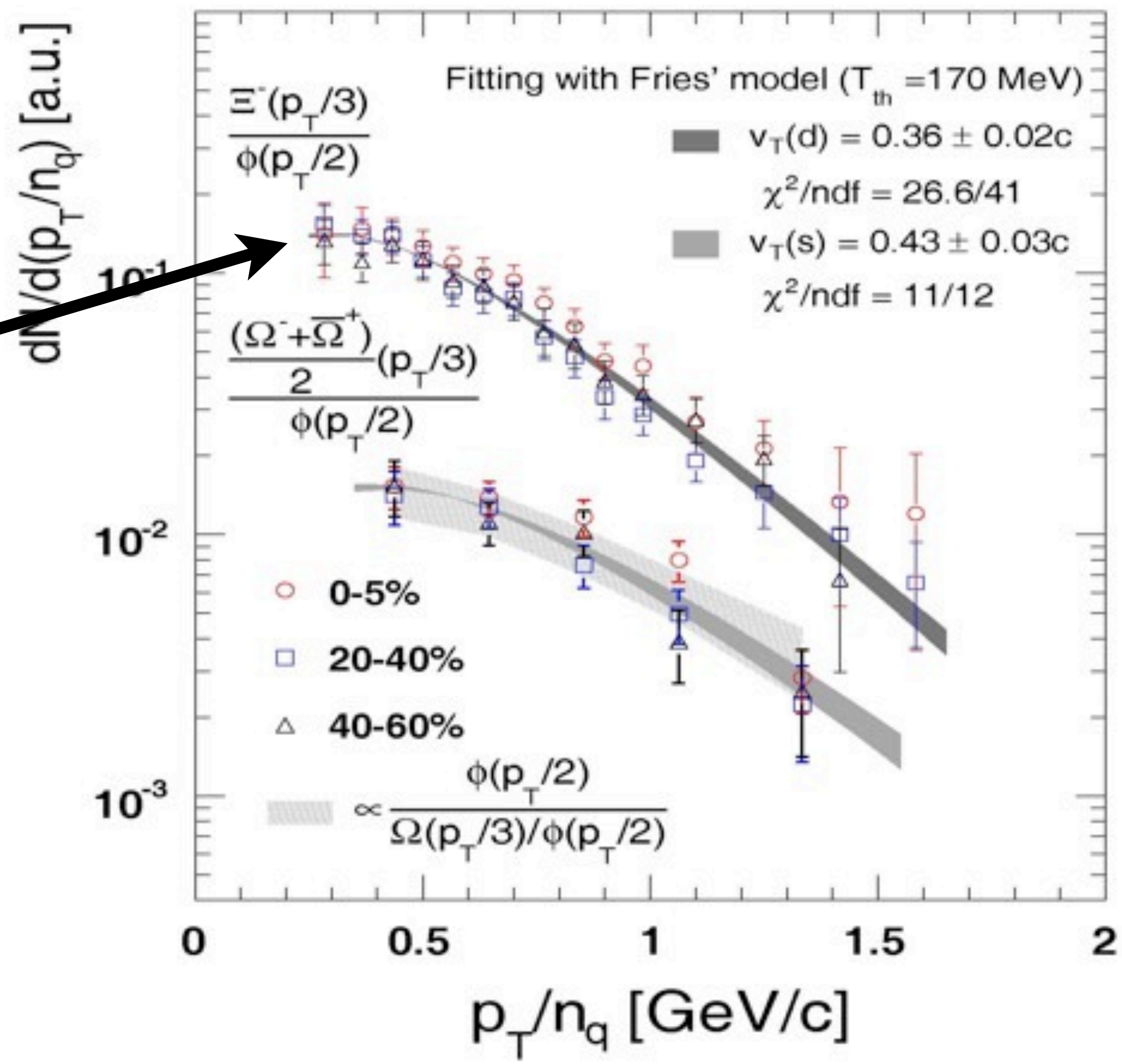
STAR/UCLA group



Quark spectroscopy

An amazing idea only experimentalists can come up with...

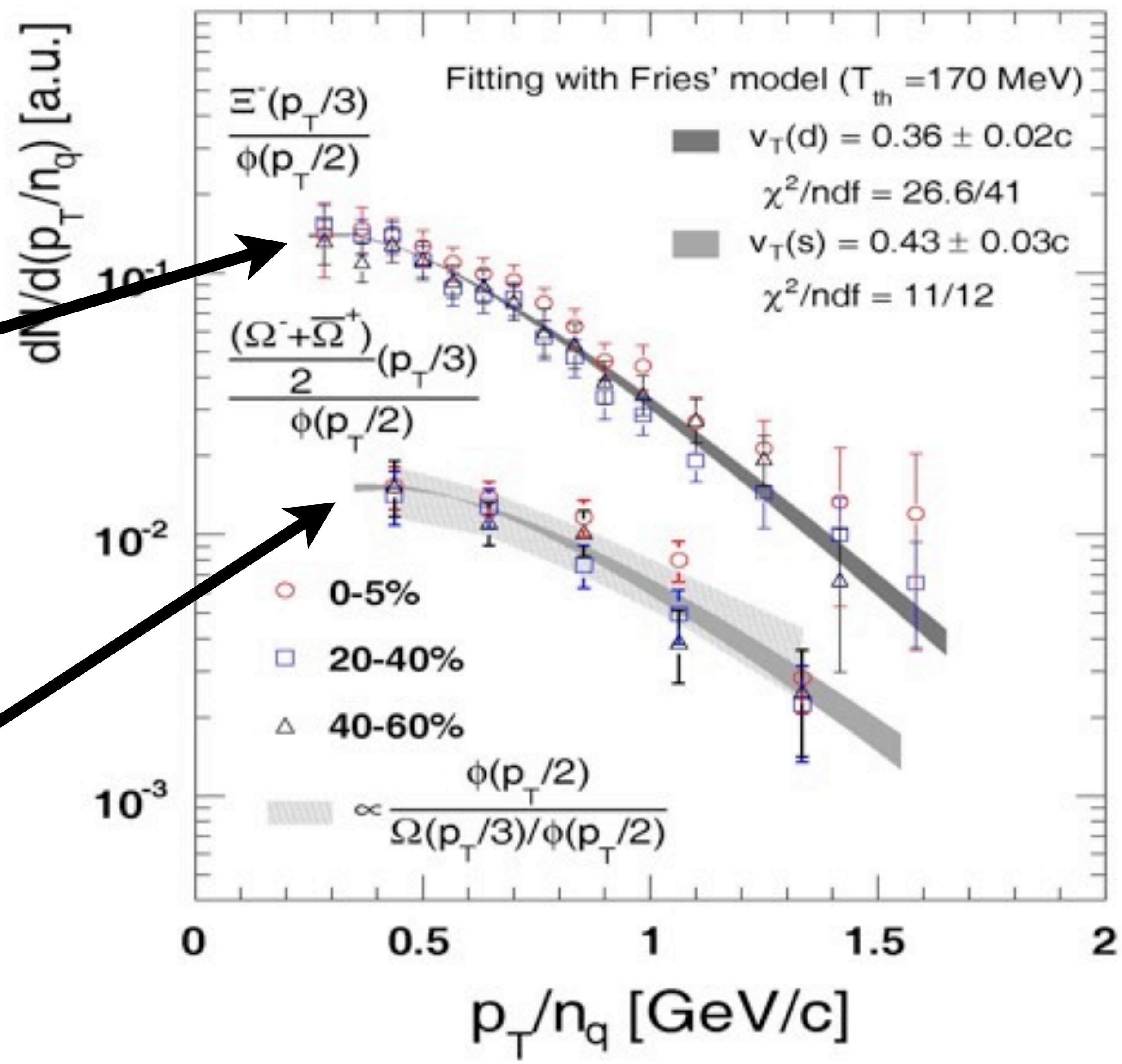
Use ratios of hadron spectra to infer valence quark spectra:

$$\Xi^- = (ssd), \quad \phi = (s\bar{s}), \quad \Omega^- = (sss)$$

$$d(p_T) = \frac{\Xi^-\left(\frac{1}{3}p_T\right)}{\phi\left(\frac{1}{2}p_T\right)}$$

$$s(p_T) = \frac{\Omega^-\left(\frac{1}{3}p_T\right)}{\phi\left(\frac{1}{2}p_T\right)} \propto \frac{\left[\phi\left(\frac{1}{2}p_T\right)\right]^2}{\Omega^-\left(\frac{1}{3}p_T\right)}$$

STAR/UCLA group



Quark spectroscopy

Instead of comments on

“Local Parity Violation”...

A temptation...

A temptation...

But good scientists are not tempted !

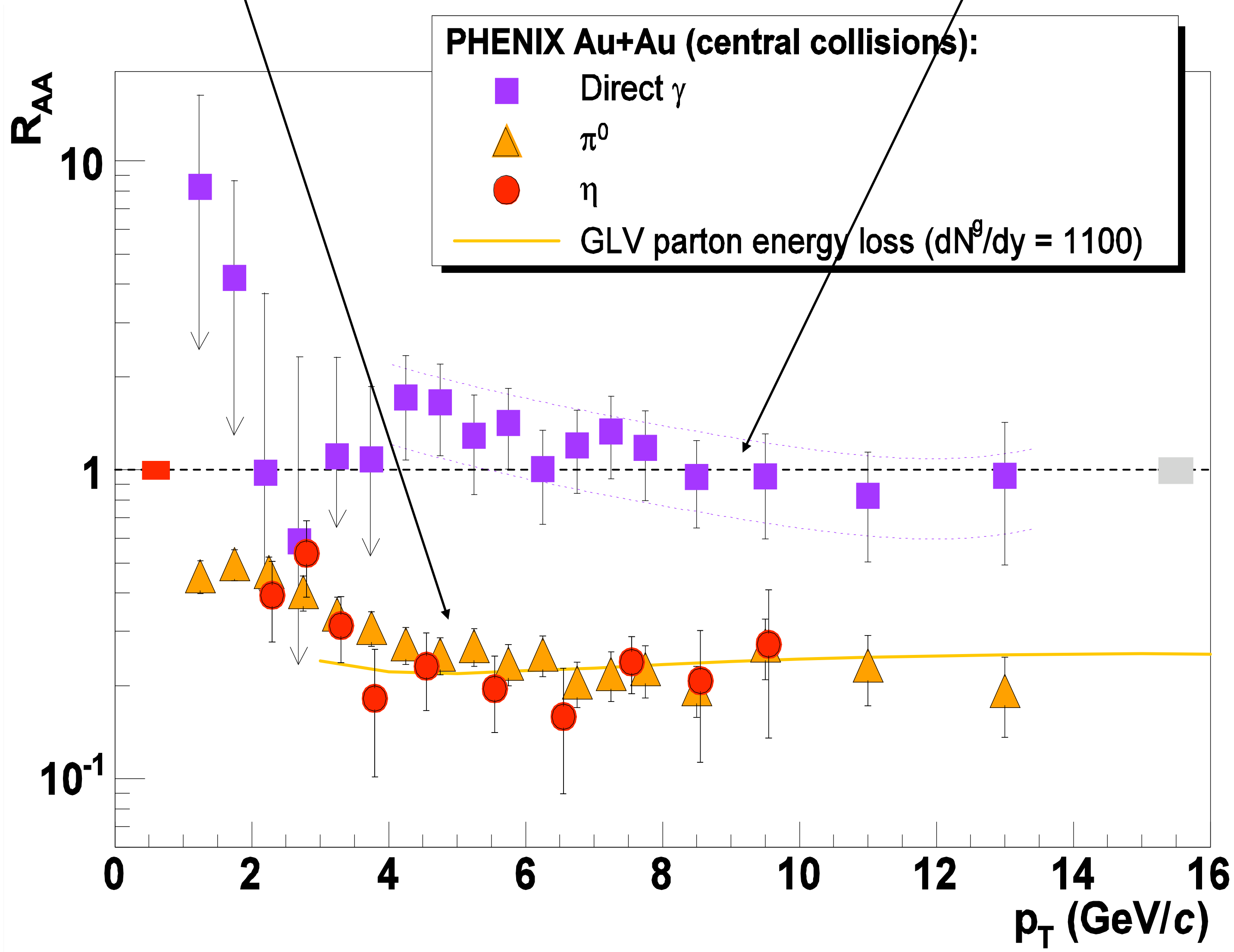
Part 4

Color Opacity

Jet quenching in Au+Au

No suppression for photons

Suppression of hadrons



Yield in A+A

$$R_{AA}(p_T) = \frac{d^2N_{AA}/dp_T dy}{T_{AA}\left(d^2\sigma_{NN}/dp_T dy\right)}$$

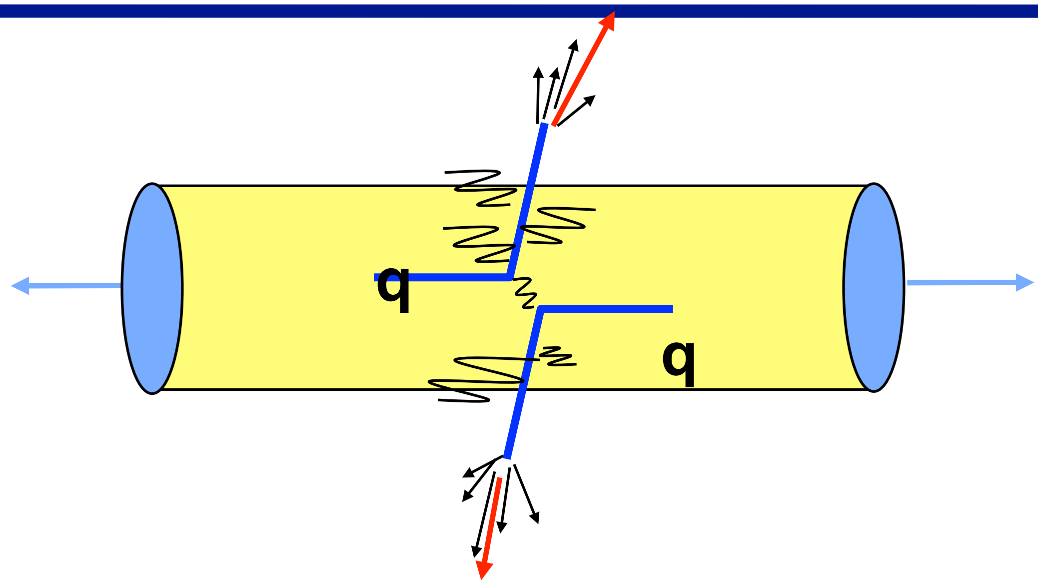
Area density of p+p coll's in A+A

Cross section in p+p coll's

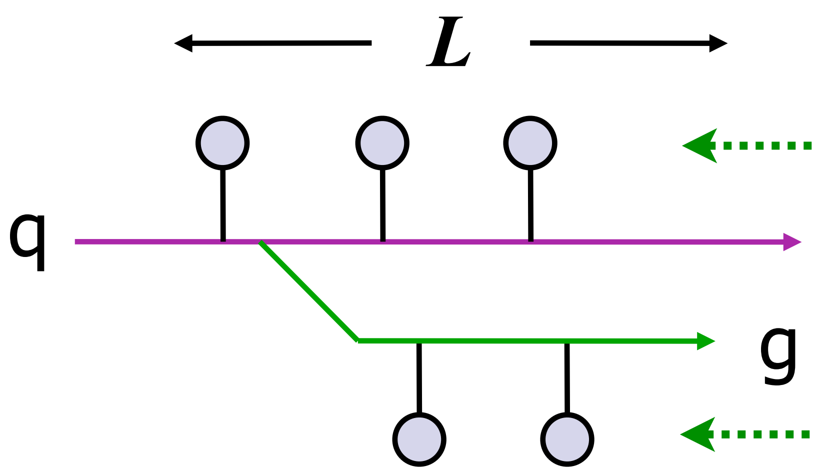
Without nuclear effects:

$R_{AA} = 1.$

Radiative energy loss



$$\Delta E \sim \rho L^2 \left\langle k_T{}^2 \right\rangle$$



Scattering centers

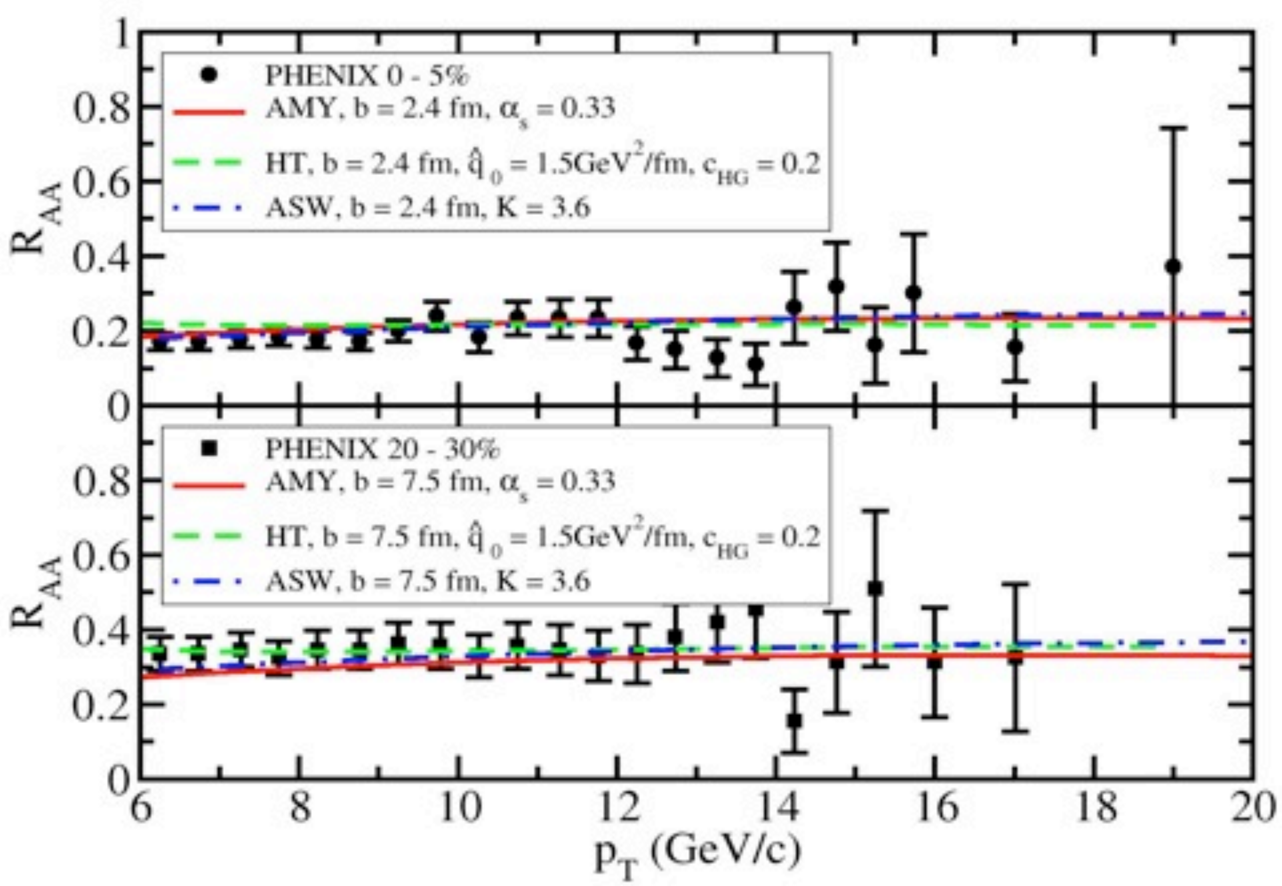
= color charges

$$\hat{q} = \rho \int q^2\, dq^2 \frac{d\sigma}{dq^2} = \int dx^- \left\langle F_i^+(x^-) F^{+i}(0) \right\rangle$$

Towards measuring $\hat{q}$

Good fits for light hadrons are obtained for all rad. energy loss models in 3-D hydrodynamics

Bass, Gale, Majumder, Nonaka, Qin, Renk & Ruppert



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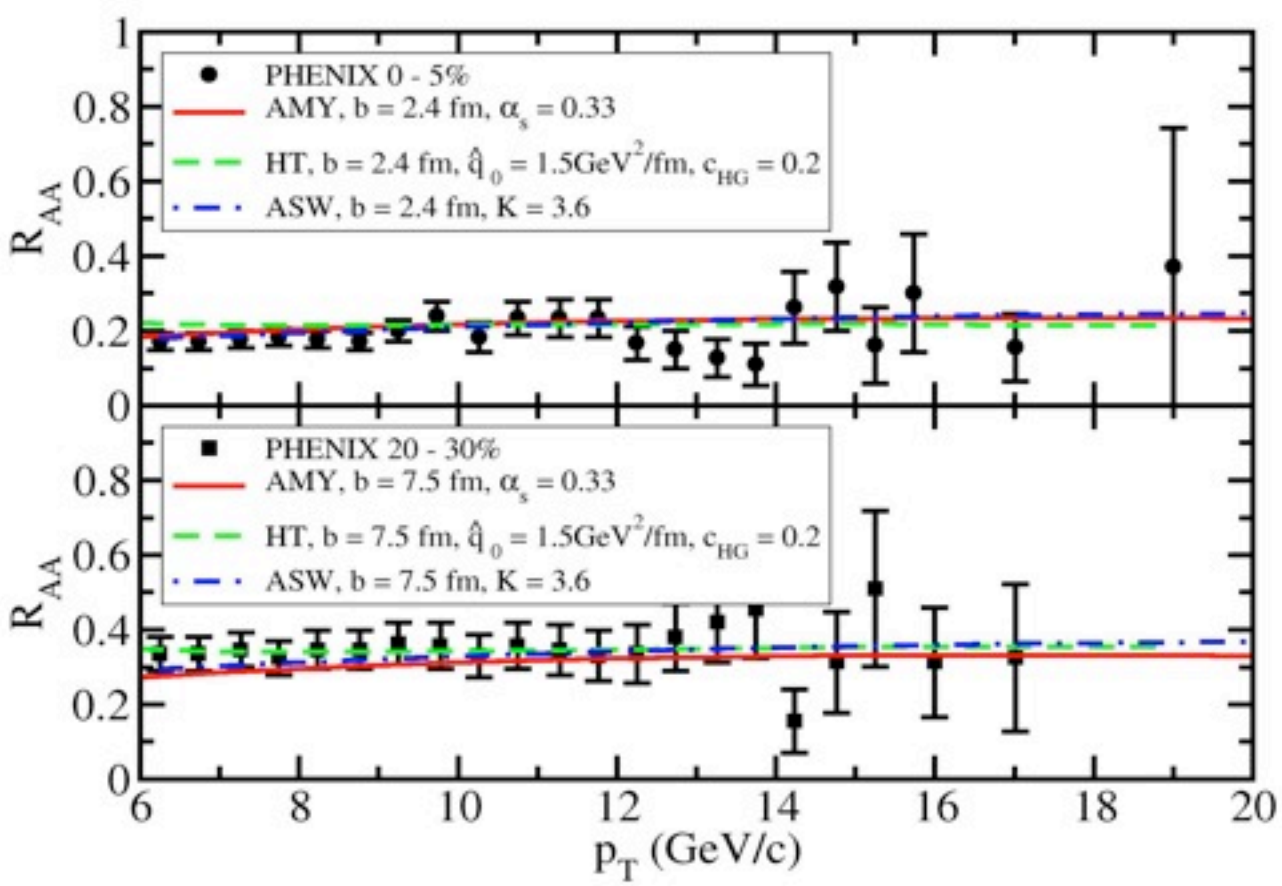

Transport parameter $\hat{q}$ deviates by more than a factor 2 between different implementations.

Caused by differences in the cut-offs in collinear approximation used in all implementations of gluon radiation.

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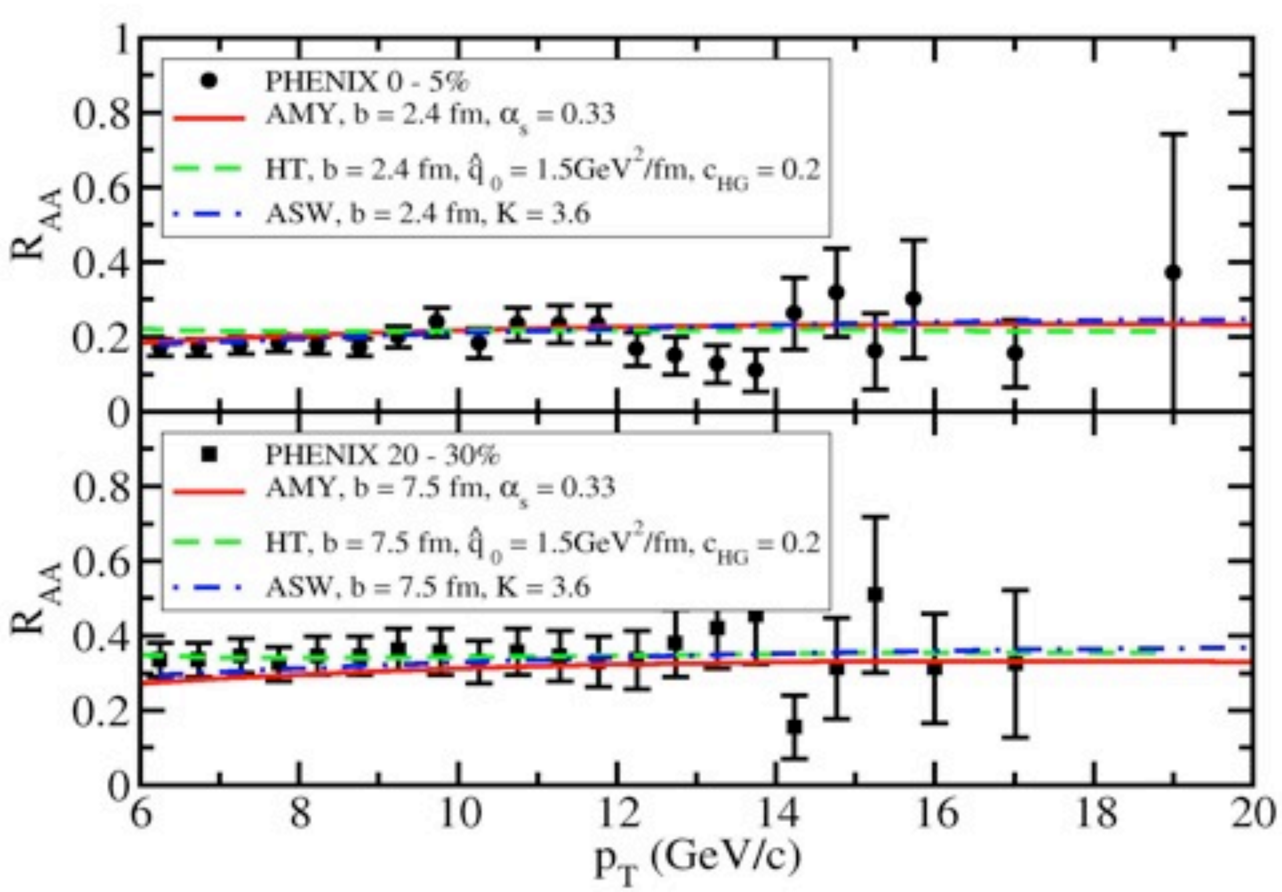


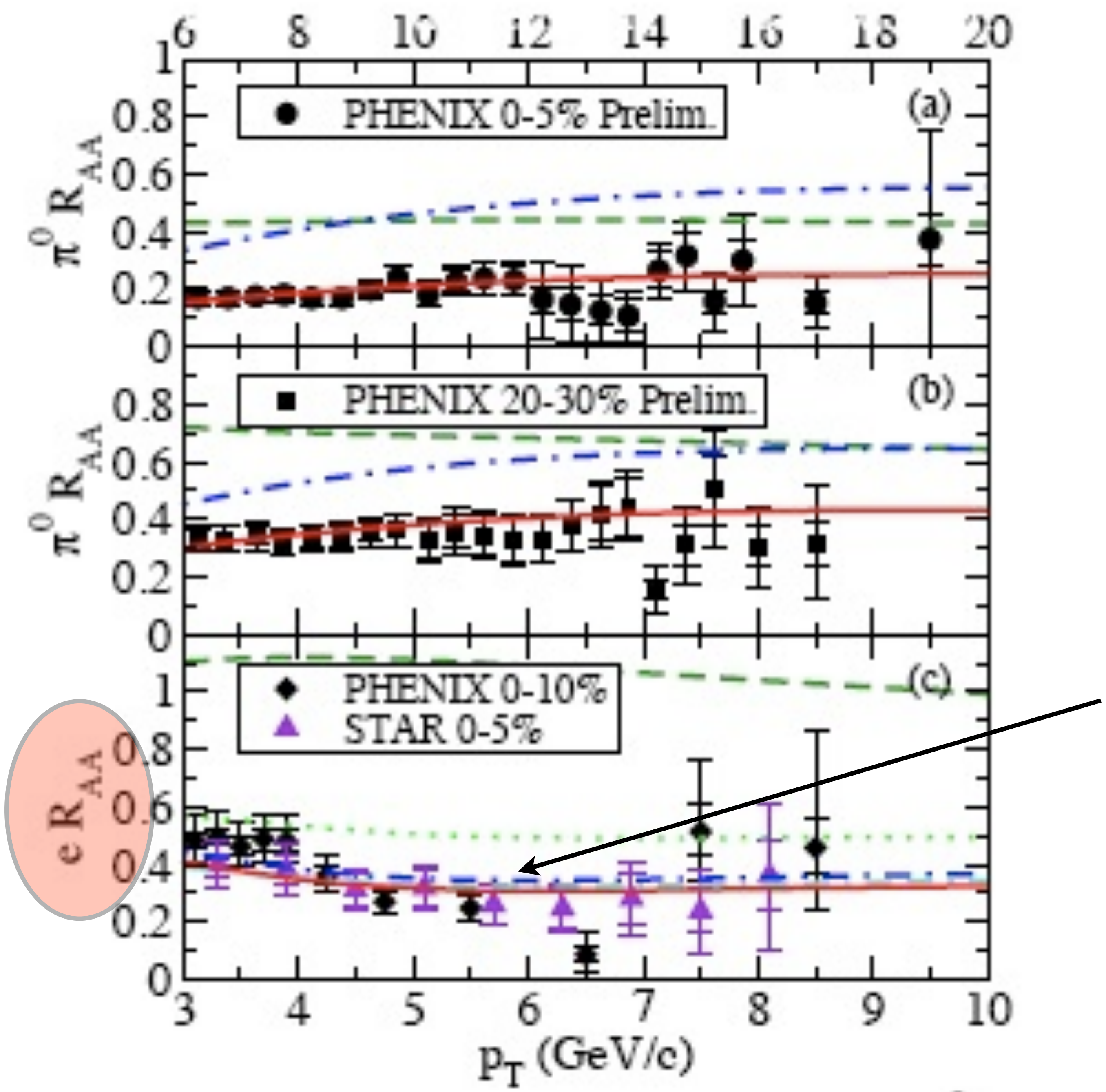
Transport parameter $\hat{q}$ deviates by more than a factor 2 between different implementations.

Caused by differences in the cut-offs in collinear approximation used in all implementations of gluon radiation.

Generalized, robust new approach needed.

The heavy quark conundrum

GY Qin & A Majumder



Heavy quark (c, b) energy loss deduced from suppression of weak decay electron spectrum

Suppression stronger than expected.

3 parameters: $\hat{q}, \quad \hat{e}, \quad \hat{e}_2$

Fit: $\dfrac{\hat{q}_c}{\hat{q}_{u/d/g}} \approx 1.1 \qquad \dfrac{\hat{q}_b}{\hat{q}_{u/d/g}} \approx 1.6$

contrary to expectations for a weakly coupled QGP.

Part 5

We now ask the question:

Is strong coupling
really necessary
for small η/s and large $\hat{q}$?

Can QCD transport be anomalous?

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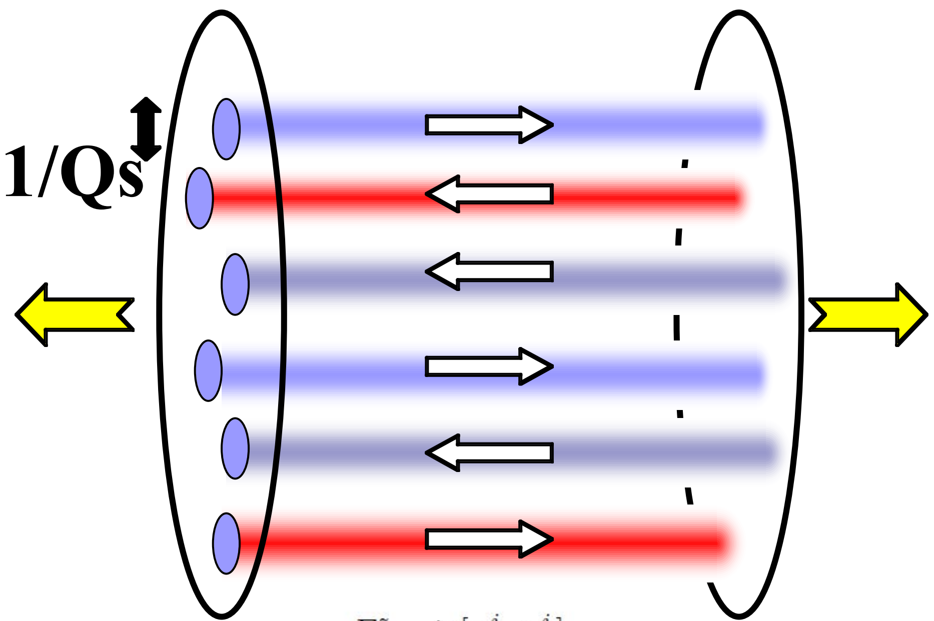
- Strong color fields in the early *glasma* exhibit Sauter and Nielsen-Olesen-type instabilities that create turbulent color fields.
- As we will see, soft color fields generate *anomalous transport coefficients*, which may dominate the transport properties of the plasma even at moderately weak coupling.

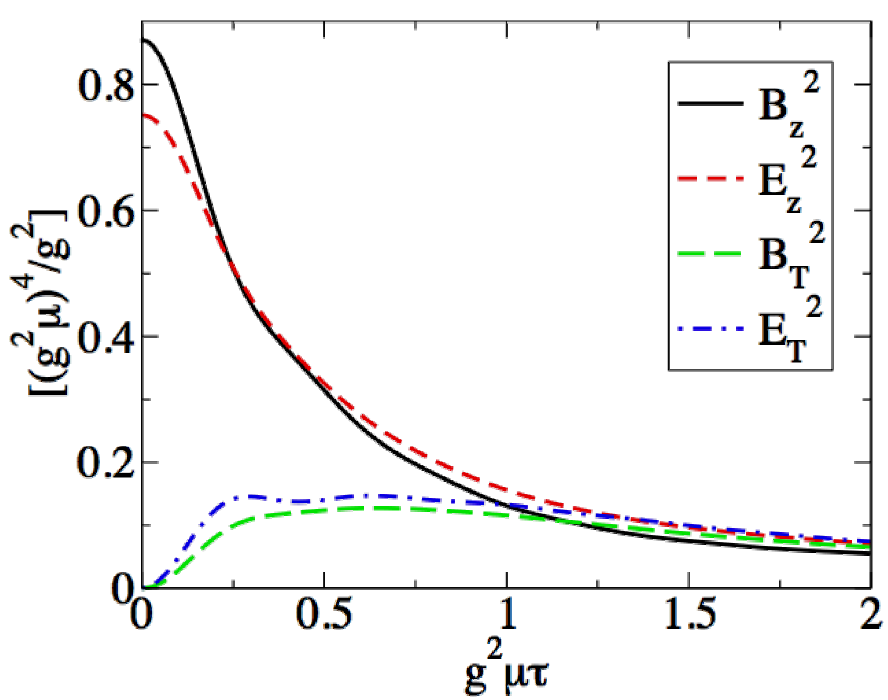
Glasma instabilities

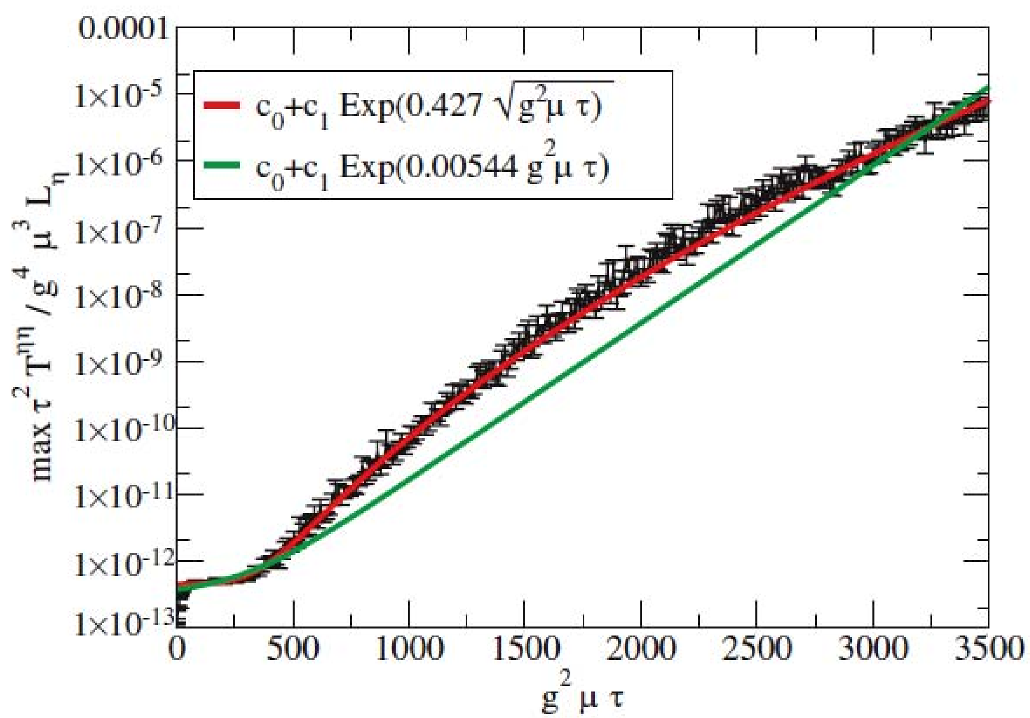


$$E^z = ig[\alpha_1^i, \alpha_2^i]$$
$$B^z = ig\epsilon^{ij}[\alpha_1^i, \alpha_2^j].$$

Nielsen-Olesen instability of longitudinal color-magnetic field (Itakura & Fujii, Iwazaki)

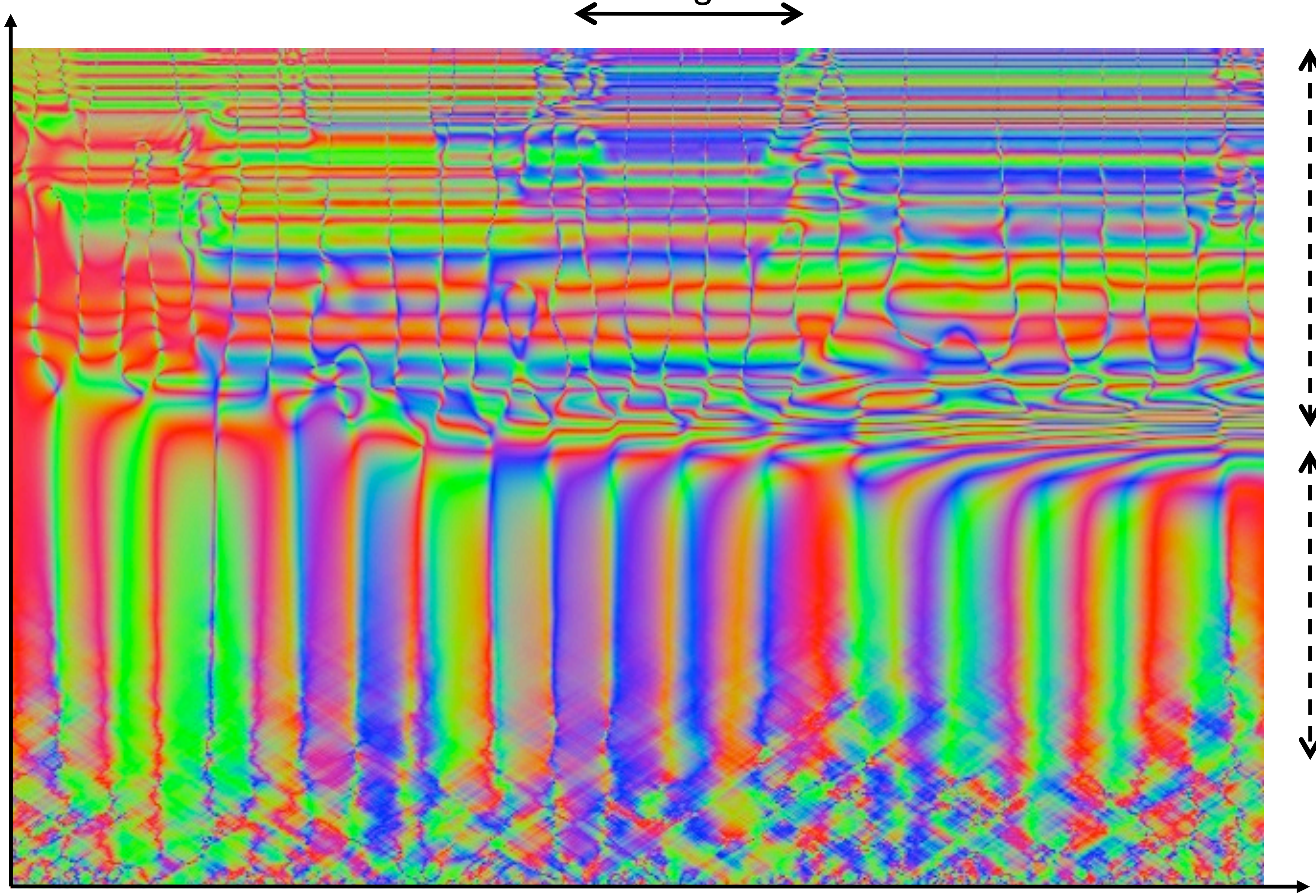
$$\frac{\partial^2 \phi}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial \phi}{\partial \tau} + \left(\frac{(k_z - gA_\eta)^2}{\tau^2} - gB_z \right)\phi = 0$$





QGP instabilities

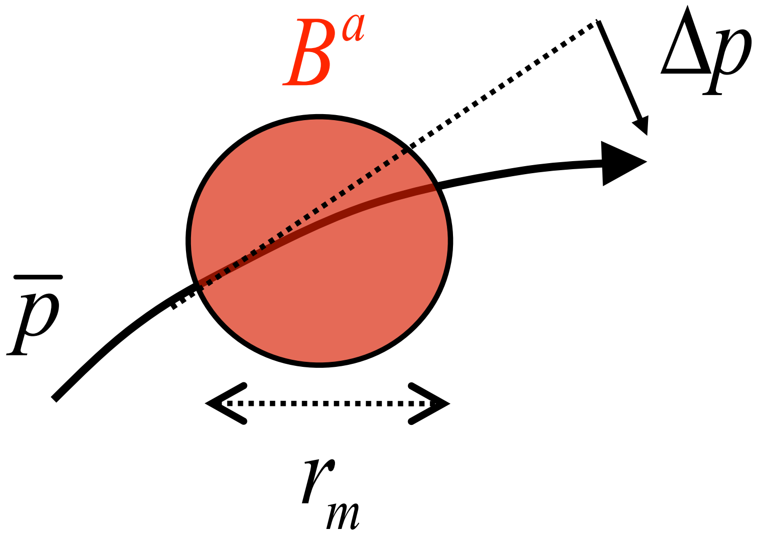
M. Strickland, hep-ph/0511212



Anomalous viscosity

Classical expression for shear viscosity:

$$\eta \approx \tfrac{1}{3} n\bar{p}\lambda_f$$



Momentum change in one coherent domain:

$$\Delta p \approx gQ^a B^a r_m$$

Anomalous mean free path in medium:

$$\lambda_f^{(A)} \approx r_m \left\langle \frac{\bar{p}^2}{(\Delta p)^2} \right\rangle \approx \frac{\bar{p}^2}{g^2 Q^2 \langle B^2 \rangle r_m}$$

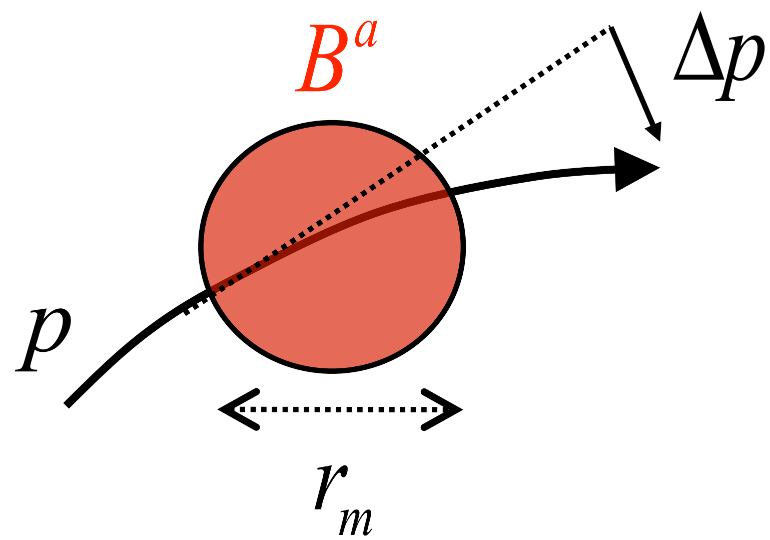
Anomalous viscosity due to random color fields:

$$\eta_A \approx \frac{n\bar{p}^3}{3g^2 Q^2 \langle B^2 \rangle r_m} \approx \frac{\frac{9}{4} sT^3}{g^2 Q^2 \langle B^2 \rangle r_m}$$

Anomalous q-hat

Jet quenching parameter:

$$\hat{q} = \frac{\langle \Delta p_T^2(L) \rangle}{L}$$



Momentum change in one coherent domain:

$$\Delta p_T = g Q^a B_\perp^a r_m$$

Anomalous jet quenching parameter:

$$\hat{q}_A = \frac{\langle \Delta p_T^2 \rangle}{r_m} = g^2 Q^2 \langle B_\perp^2 \rangle r_m$$

Relation to anomalous shear viscosity:

$$\frac{\eta_A}{s} \approx \frac{T^3}{\hat{q}_A}$$

Special case of general relation between η/s and q^ (A. Majumder, BM & Wang, PRL 99, 192301 ('07).

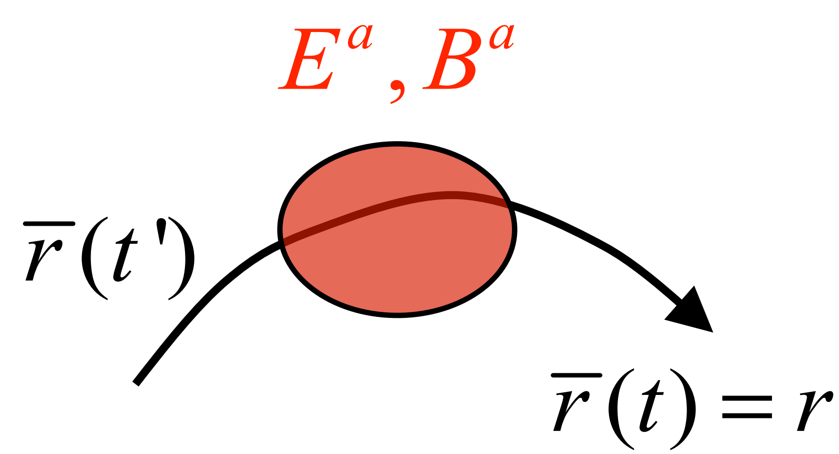
Turbulence ⇔ p-Diffusion

Vlasov-Boltzmann transport of thermal partons:

$$\left[\frac{\partial}{\partial t}+\frac{p}{E_p}\cdot\nabla_r+F\cdot\nabla_p\right]f(r,p,t)=C[f]$$

with Lorentz force

$$F=gQ^a\left(E^a+\mathrm{v}\times B^a\right)$$



Assuming E, B random $\Rightarrow$ Fokker-Planck eq:

$$\left[\frac{\partial}{\partial t}+\frac{p}{E_p}\cdot\nabla_r-\nabla_p\cdot D(p)\cdot\nabla_p\right]\bar{f}(r,p,t)=C\left[\bar{f}\right]$$

with diffusion coefficient

$$D_{ij}(p)=\int_{-\infty}^{t}dt'\left\langle F_i\left(\bar{r}(t'),t'\right)F_j\left(r,t\right)\right\rangle.$$

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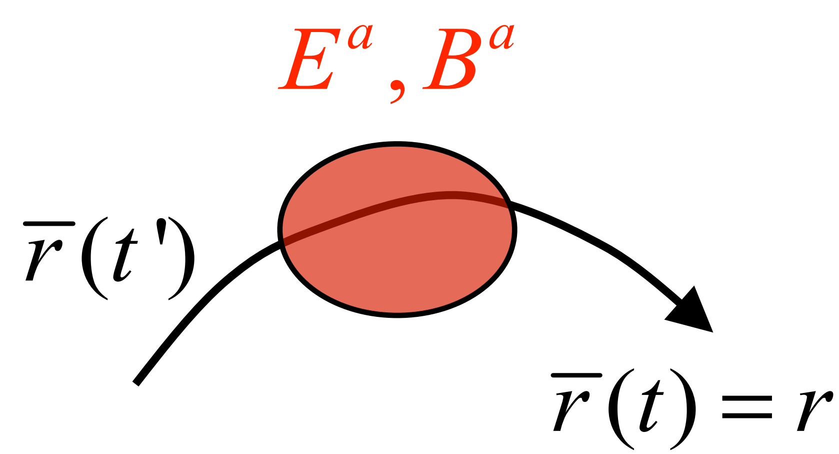

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$$D_{ij}(p)=\int_{-\infty}^{t}dt'\left\langle F_i\left(\bar{r}(t'),t'\right)F_j\left(r,t\right)\right\rangle.$$

Diffusion is dominated by chromo-magnetic fields:

$$\int dt'\left\langle B(t')B(t)\right\rangle\equiv\left\langle B^2\right\rangle\tau_m$$

Weibel regime

Take moments of $\left[\frac{\partial}{\partial t}+\frac{p}{E_p}\cdot\nabla_r-\nabla_p\cdot D(p)\cdot\nabla_p\right]\bar{f}(r,p,t)=C\left[\bar{f}\right]$ with p_z^2

$$\frac{1}{\eta}=O(1)\frac{N_c}{N_c^2-1}\frac{g^2\langle B^2\rangle\tau_m}{sT^3}+O\left(10^{-2}\right)\frac{g^4\ln g^{-1}}{T^3}\equiv\frac{1}{\eta_A}+\frac{1}{\eta_C}$$

Self-consistency $\Rightarrow$ $\frac{\eta_A}{s}\sim\left(\frac{T}{g^3|\nabla u|}\right)^{1/2}$ compare with $\frac{\eta_C}{s}\sim\frac{1}{g^4\ln g^{-1}}$

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$$\frac{\eta_A}{s} \sim \left(\frac{T}{g^3|\nabla u|}\right)^{1/2}$$

compare with

$$\frac{\eta_C}{s} \sim \frac{1}{g^4 \ln g^{-1}}$$

Anomalous shear viscosity dominates over *collisional shear viscosity* at fixed ∇u in the limit $g \rightarrow 0$.

Glasma regime

In the *glasma*, most of the energy density is in the form of *color fields*:

Anomalous transport dominates over Boltzmann (collision) transport.

$$\eta_A \approx \frac{N_c^2 - 1}{15\pi^2 C_2 g^2 \left\langle \mathcal{E}^2 + \mathcal{B}^2 \right\rangle \tau_m} \int_0^\infty dp\, p^5 f(p) \approx \frac{\left(N_c^2 - 1\right) Q_s^2}{C_2 g^2 \tau_m} \cdot \frac{\varepsilon_{\rm part}}{\varepsilon_{\rm field}}$$

Anomalous jet quenching:

$$\hat{q}_A \approx \frac{C_2 g^2 \left\langle \mathcal{E}^2 + \mathcal{B}^2 \right\rangle \tau_m}{N_c^2 - 1} \approx \frac{g^2 \varepsilon_{\rm field}}{Q_s} \approx \frac{Q_s^3}{(Q_s \tau)} \approx \frac{10\ \mathrm{GeV}^2/\mathrm{fm}}{Q_s \tau}$$

In line with estimates of q^ ~ 2 - 4 GeV^2/fm from fits to data.

Caveat explorator

Caveat explorator

QCD
makes it hard to
distinguish between
"queen snakes"
and
"old tree branches"

Connecting jets with the medium

Hard partons probe the medium via the density of colored scattering centers:

$$\hat{q} = \rho \int q^2 dq^2 \left(d\sigma / dq^2 \right) \sim \int dx^- \left\langle F^{\perp +}(x^-) F_\perp^+(0) \right\rangle$$

If kinetic theory applies, thermal gluons are quasi-particles that experience the same medium. Then the shear viscosity is:

$$\eta \approx \frac{1}{3} \rho \left\langle p \lambda_f(p) \right\rangle = \frac{1}{3} \left\langle \frac{p}{\sigma_{tr}(p)} \right\rangle$$

In QCD, small angle scattering dominates: $\sigma_{tr}(p) \approx \dfrac{2\hat{q}}{\langle p \rangle^2 \rho}$

With $\langle p \rangle \sim 3T$ and $s \approx 3.6\rho$ (for gluons) one finds:

$$\frac{\eta}{s} \approx \frac{T^3}{\hat{q}} \ln \sqrt{\frac{E}{m_D}}$$

A. Majumder, BM, X-N. Wang, PRL 99 (2007) 192301

Example: N = 4 SYM

Strong coupling:

$$\frac{\eta}{s}=\frac{1}{4\pi}\left[1+\frac{135}{16\sqrt{2}}\zeta(3)\left(g^2N_c\right)^{-3/2}+\cdots\right]$$

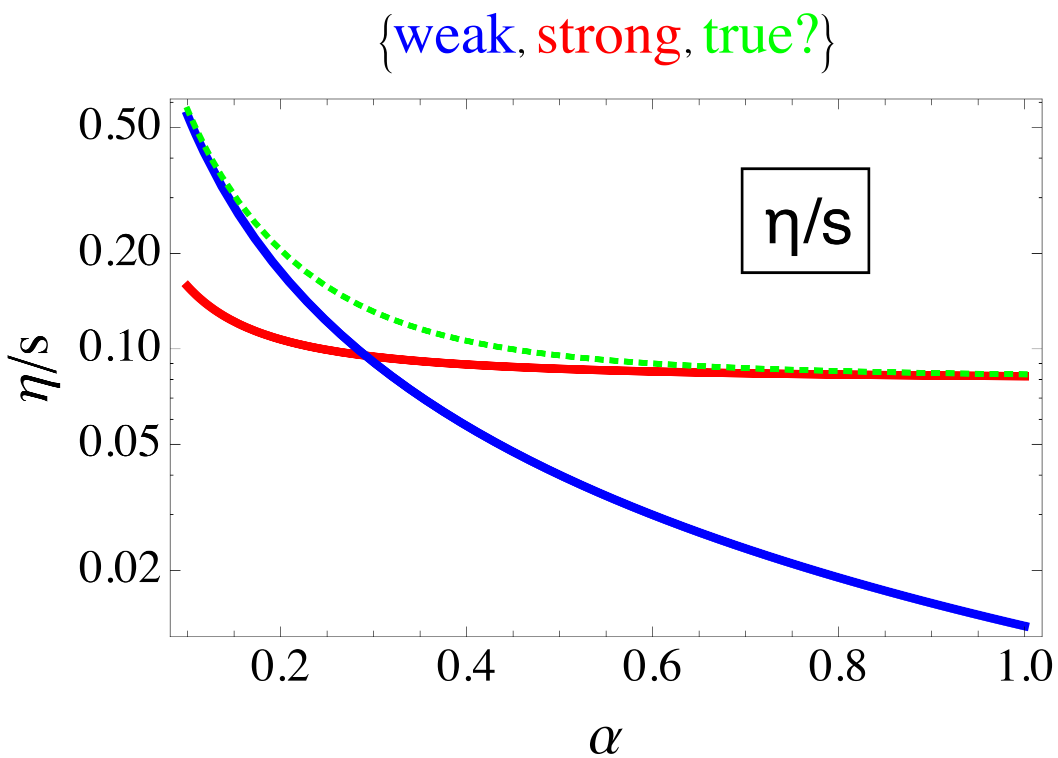
$$\frac{\hat{q}}{T^3}=\pi^{3/2}\frac{\Gamma(3/4)}{\Gamma(5/4)}\sqrt{g^2N_c}$$

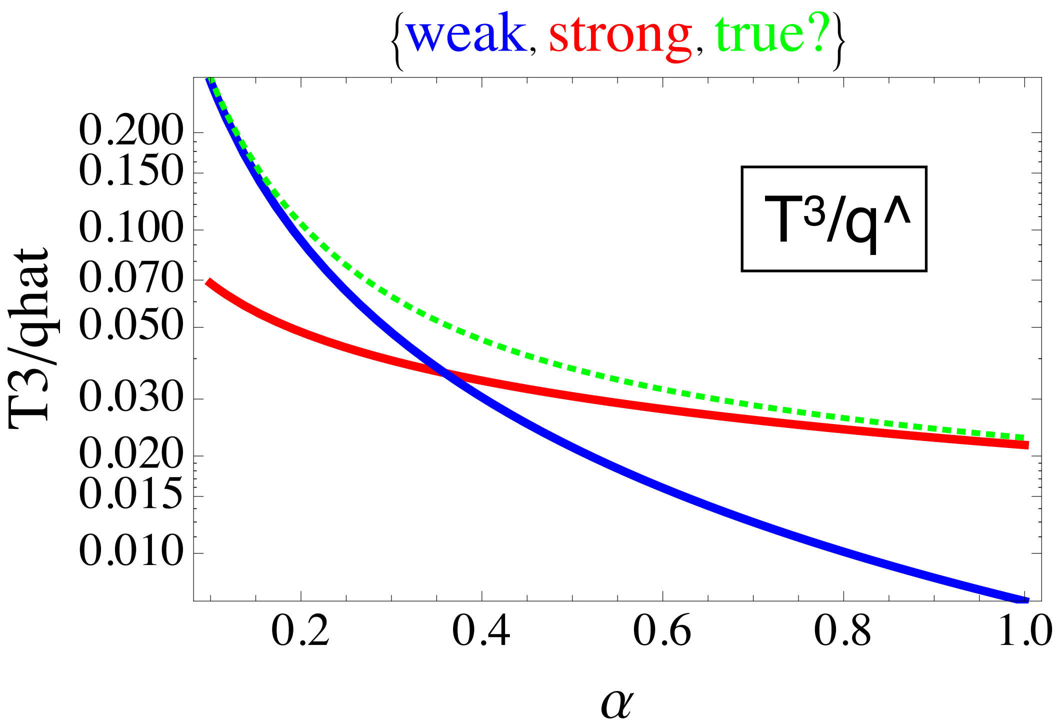
Weak coupling:

$$\frac{\eta}{s}=\frac{6.174}{\left(g^2N_c\right)^2\ln\left(2.36/\sqrt{g^2N_c}\right)}$$

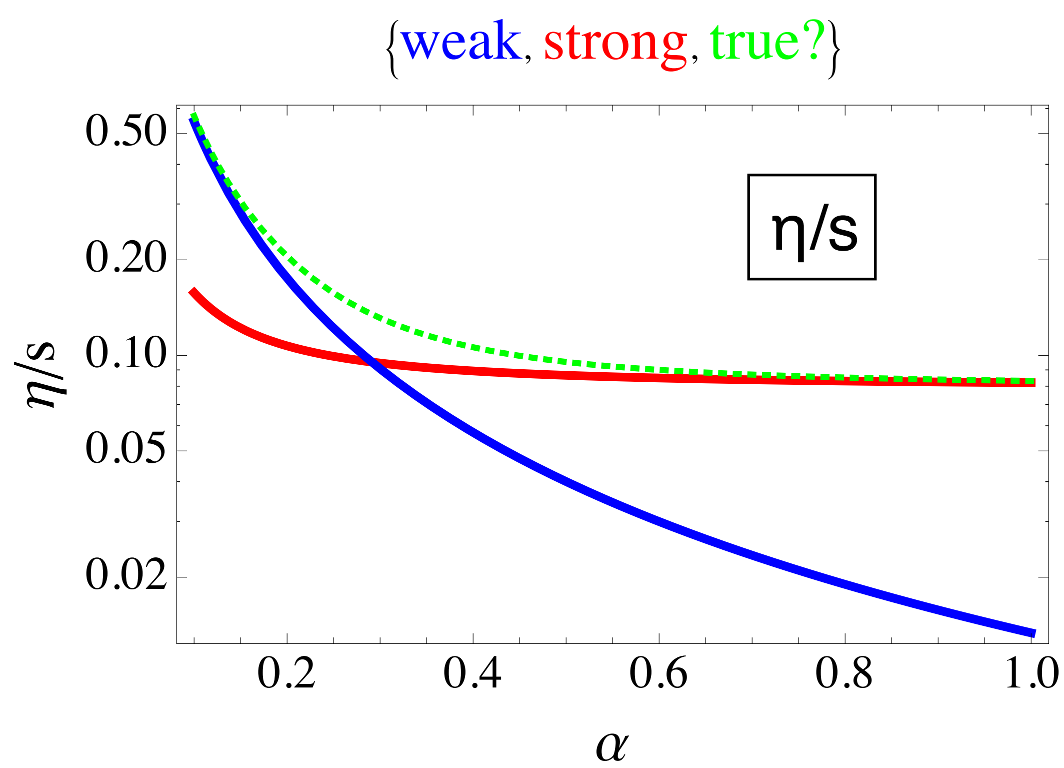
$$\frac{\hat{q}}{T^3}\approx\frac{\left(g^2N_c\right)^2}{\pi}\ln\sqrt{\frac{E}{m_D}}$$

Comparison

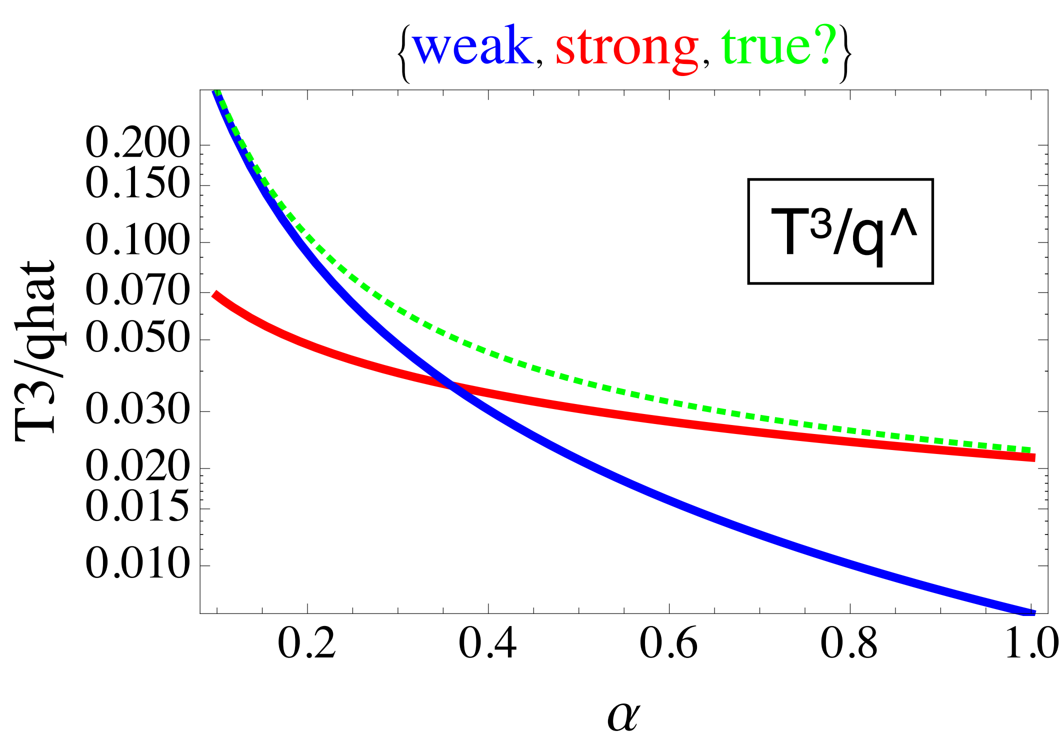

{weak, strong, true?}
η/s
0.50
0.20
0.10
0.05
0.02
η/s
0.2
0.4
0.6
0.8
1.0
α

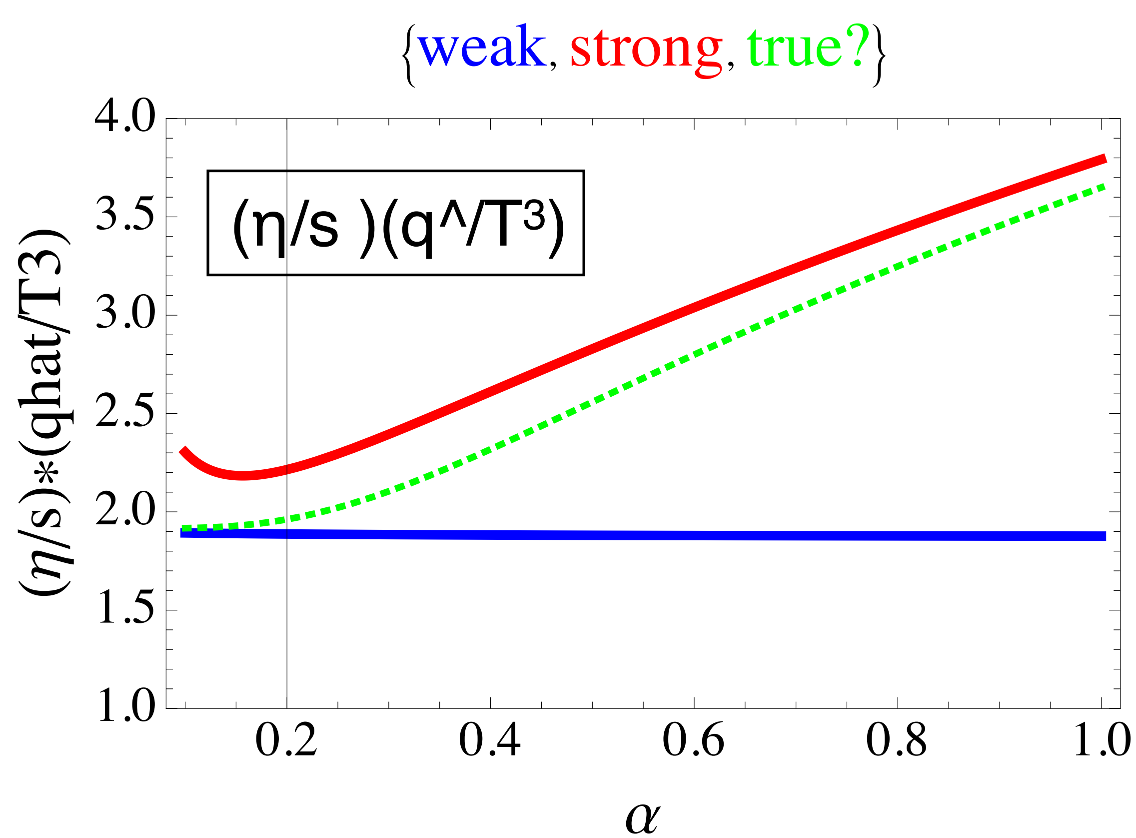

{weak, strong, true?}
T³/q^
0.200
0.150
0.100
0.070
0.050
0.030
0.020
0.015
0.010
T3/qhat
0.2
0.4
0.6
0.8
1.0
α

Comparison



Blue line is independent of α_s, but slowly moves up and down with parton energy;
Red line depends on α_s, but is independent of parton energy (?)





Observables revisited

Which properties of hot QCD matter can we hope to determine from relativistic heavy ion data ?

$T_{\mu\nu} \quad \Leftrightarrow \quad \varepsilon, p, s$ **Equation of state**: spectra, collective flow

$c_c^2 = \partial p / \partial \varepsilon$ **Speed of sound**: multiparticle correlations

$$\eta = \frac{1}{T} \int d^4x \left\langle T_{xy}(x) T_{xy}(0) \right\rangle$$

Shear viscosity: anisotropic collective flow

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \left\langle F^{a+i}(y^-) F_i^{a+}(0) \right\rangle$$

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Momentum/energy diffusion:
parton energy loss
modified jet fragmentation

$$m_D = -\lim_{|x| \to \infty} \frac{1}{|x|} \ln \left\langle E^a(x) E^a(0) \right\rangle$$

Color screening: Quarkonium states

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Ready for a serious attempt

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Serious theoret. develop-ments needed

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Color screening: Quarkonium states

Challenge to students

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Are you ready to beat Lucy's deadline and prove that *the world was once* (13.7 billion years ago) *a perfect fluid ?*

Do I hear an emphatic YES *?*